ANALOG COMMUNICATION
Course Code: 328452(28)

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Course Contents

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UNIT-II ANGLE MODULATION
UNIT-III MATHEMATICAL REPRESENTATION OF NOISE
UNIT-IV NOISE IN AM SYSTEMS
UNIT-V NOISE IN ANGLE MODULATED SYSTEMS

Text Books:

1. Principles of Communication Systems, Taub and Schilling, 2nd Edition., Tata McGraw Hill.(Unit-I,II,III,IV,V)
2. Electronic Communication Systems, George F Kennedy, Tata McGraw Hill. (Unit-I, II)
3. Communication Systems, Simon Haykins, Wiley India

Reference Books:

1. Communication Systems Engineering, Proakis, 2nd Edition, Pearson Education.
2. Modern Digital and Analog Communication, B.P. Lathi, Oxford University Press.
3. Communication Systems (Analog and Digital), Singh and Sapre, 2nd Edition, Tata McGraw Hill

UNIT-II Angle Modulation:
Phase & frequency modulation

- Angle modulation: Phase & frequency modulation
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- Phase and frequency deviation
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UNIT-II Angle Modulation:

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- Bandwidth required for a Gaussian modulated WBFM signal
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- Frequency multiplication, Frequency multiplication applied to FM signals,
- FM demodulators
- Approximately compatible SSB systems
- Stereophonic FM broadcasting.
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Introduction

- Angle modulation is another significance process of message transmission.
- The frequency modulation has an important advantage over AM i.e. interference due to noise is considerably reduced.
- This advantage of noise immunity is at the cost of increased bandwidth ,hence a less no. of channel can be accommodated in a given frequency band.
- Angle modulation is the process of varying the total phase angle (frequency or phase) of a carrier wave in accordance with the instantaneous value of the message signal, keeping amplitude of the carrier constant.

Introduction……..

- An unmodulated carrier is given by
  $\varphi(t) = A\cos(\omega_c t+\Theta_0)$……………….Eq.(1)
  $\varphi(t) = A\cos(\psi)$ ……………….Eq.(2)
- where ,$\psi = (\omega_c t+\Theta_0)$ is the total angle of the carrier wave.
- The eqution No. (2) $\varphi(t) = A\cos(\psi)$ may be considered as the real part of a rotating phasor = $Ae^{j\psi}$

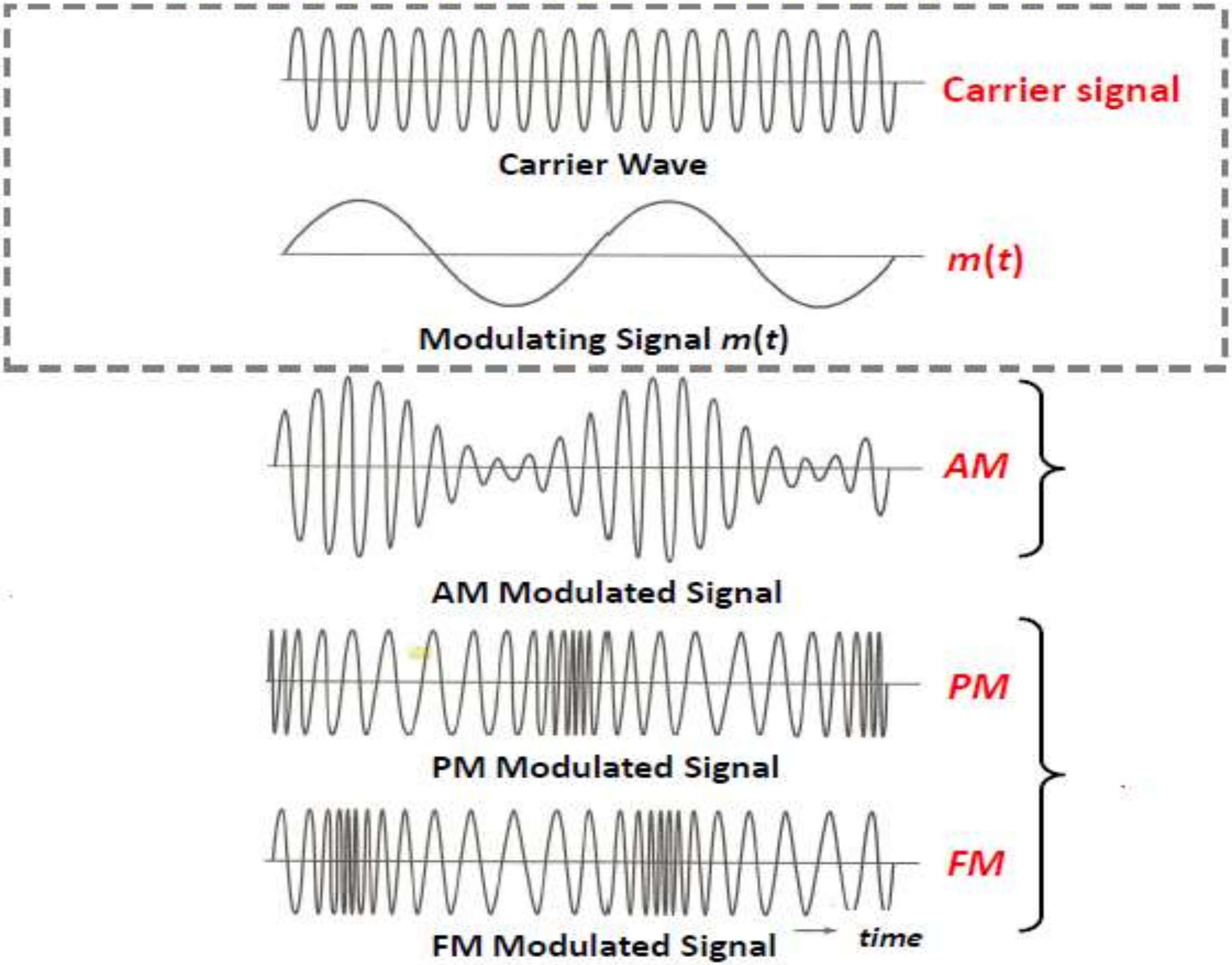
  $\varphi(t) = \mathrm{Re}[Ae^{j\psi}] = A\,\mathrm{Re}[\cos\psi + j\sin\psi] = A\cos(\psi)$
- The phasor rotates at a constant angular velocity ω_c provided Θ_0 is independent of time.
- The constant angular velocity ω_c of phasor is related to its total angle $\psi(t)$ of the carrier wave is derived below :
  $\psi = (\omega_c t+\Theta_0)$
- Differentiating the above equation , $(d\psi/dt) = \omega_c$

.

Introduction………..

- This derivative is constant with time an unmodulated carrier. The time dependent angular velocity is called instantaneous angular velocity, denoted by $(d\psi/dt)= \omega_i$
- $\psi=\int \omega_i \, dt$, ω_i is time dependent
- Angle modulation is classified into two types such as
  - **Frequency modulation (FM):**In frequency modulation the amplitude and phase of carrier is kept constant and the frequency of the carrier signal is varied in accordance with the instantaneous value of modulating signal.
  - **Phase modulation (PM):** In phase modulation the amplitude and frequency of carrier is kept constant and the phase of the carrier signal is varied in accordance with the instantaneous value of modulating signal.
- Application :
  - Commercial radio broadcasting
  - Television sound transmission
  - Two way mobile radio
  - Cellular radio
  - Microwave and satellite communication system

Illustrating AM, PM and FM Signals



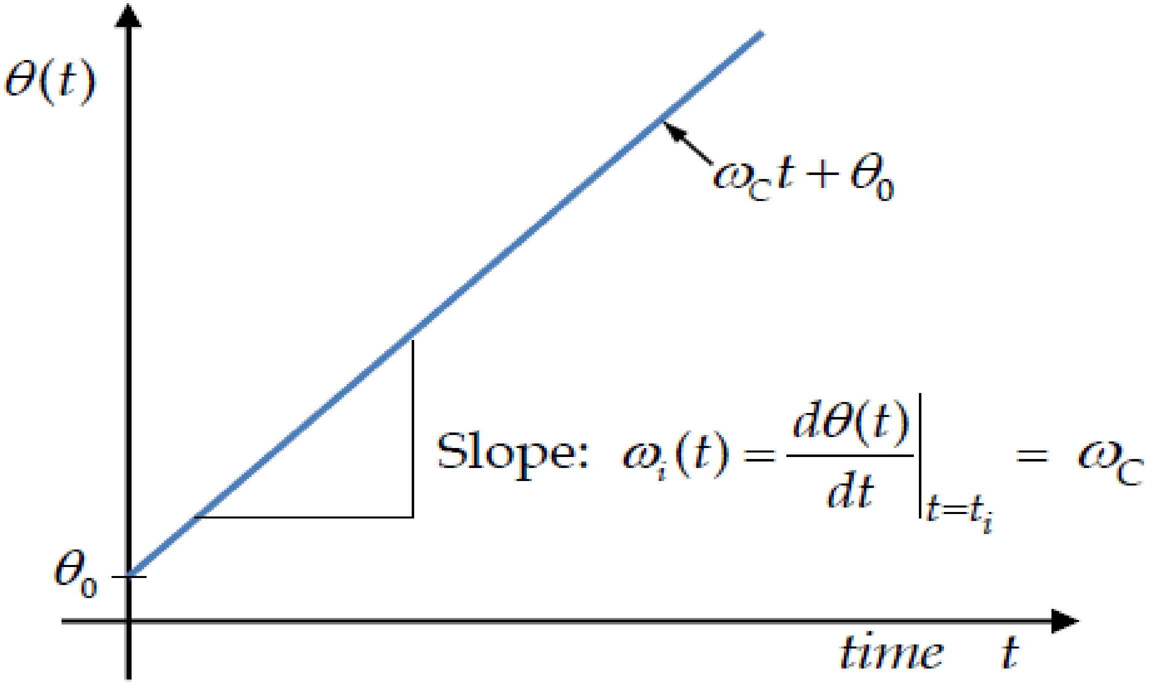
Phase & frequency modulation

Phase-Frequency Relationship When Frequency is Constant

$$\varphi(t) = A\cos(\theta(t))$$

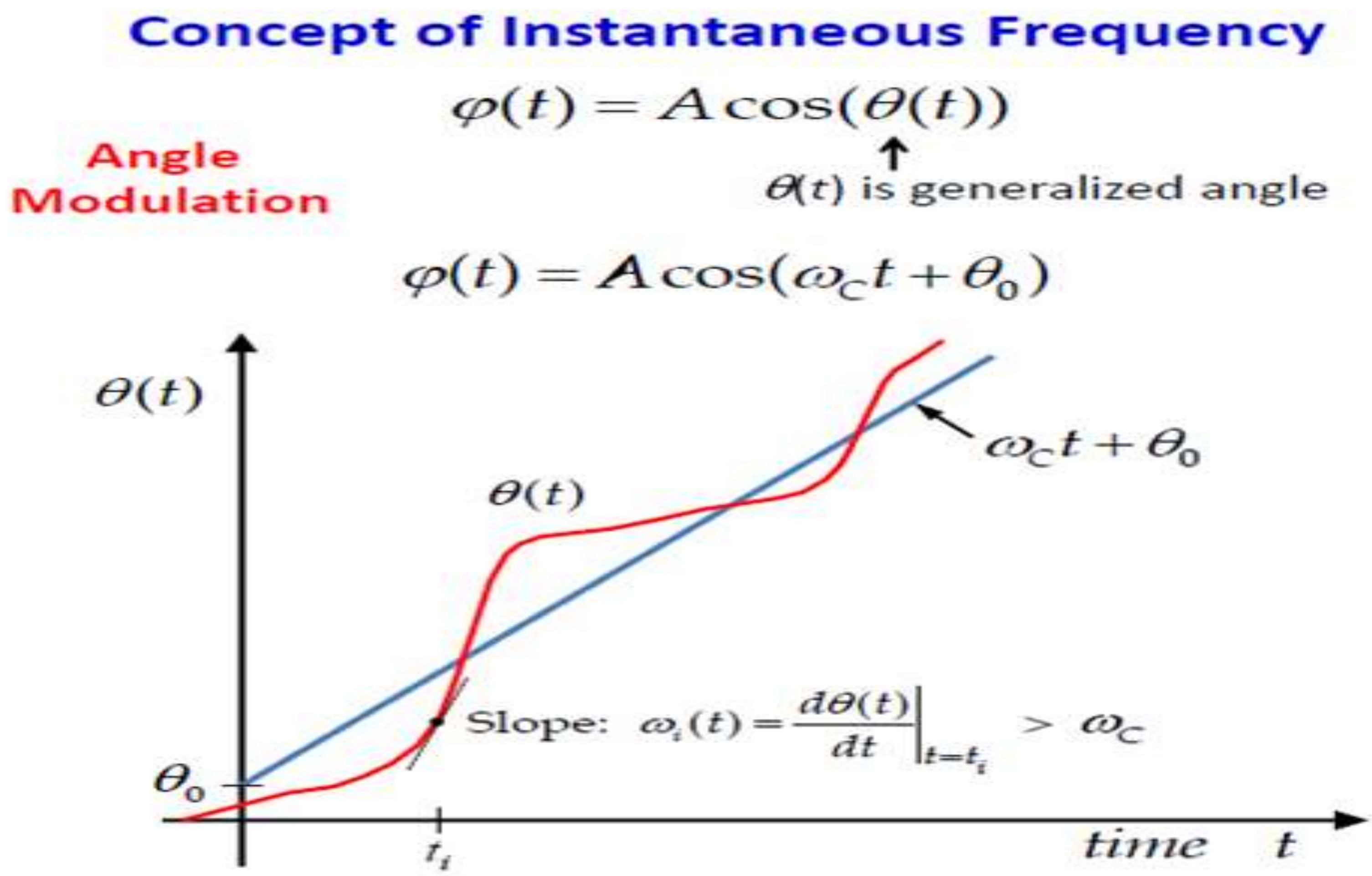
$\theta(t)$ is generalized angle

$$\varphi(t) = A\cos(\omega_C t + \theta_0)$$

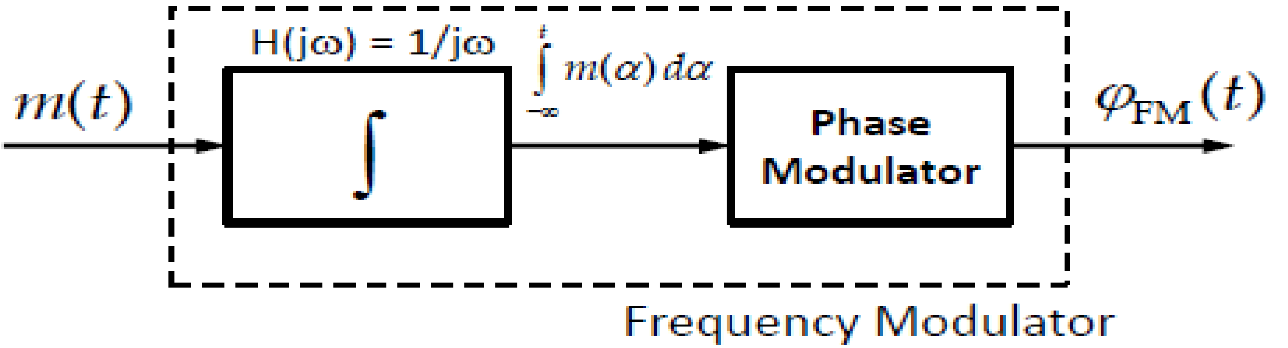


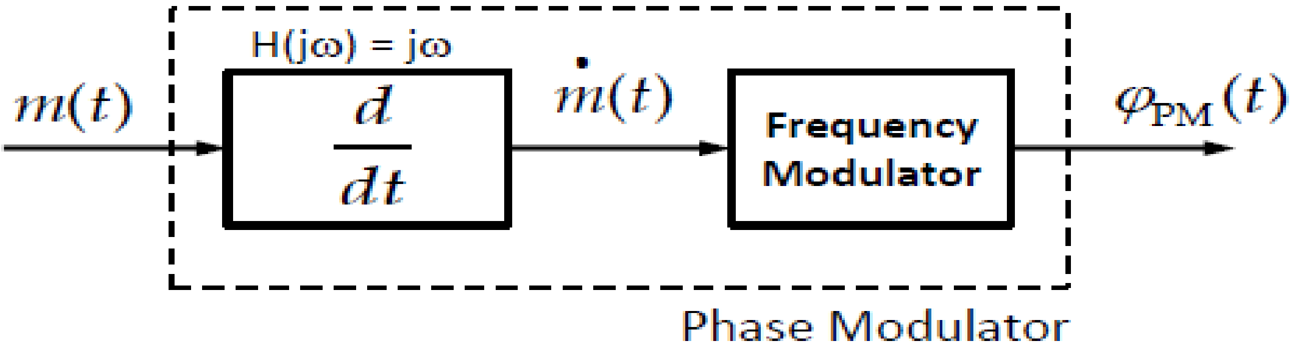
The time dependent angular velocity ω_i of phasor provides a time varying instantaneous frequency f_i of carrier wave φ (t).

The frequency of carrier wave changes from one cycle to another .



RelationShip between PM and FM





We require that H(jω) be a reversible (or invertible) operation so that m(t) is recoverable.

Frequency Modulation (FM)

But in frequency modulation the instantaneous angular frequency ω_i varies linearly with the modulating signal $m(t)$,

$$\omega_i = \omega_C + k_f m(t)$$

$$\theta(t) = \int_{-\infty}^{t} (\omega_C + k_f m(\alpha))\, d\alpha = \omega_C t + k_f \int_{-\infty}^{t} m(\alpha)\, d\alpha$$

k_f is frequency-deviation (sensitivity) constant. Units: radians/volt-sec.

Then

$$\varphi_{FM}(t) = A\cos\left(\omega_C t + k_f \int_{-\infty}^{t} m(\alpha)\, d\alpha \right)$$

FM and PM are very much related to each other.
In PM the angle is directly proportional to $m(t)$.
In FM the angle is directly proportional to the integral of $m(t)$, *i.e.*, $\int m(t)dt$

Phase Modulation (PM)

$$\theta(t) = \omega_C t + \theta_0 + k_p m(t) \qquad \text{Generally we let } \theta_0 = 0.$$

$Let\ \theta_0 = 0$

$$\varphi_{PM}(t) = A\cos(\omega_C t + k_p m(t))$$

The instantaneous angular frequency (in radians/second) is

$$\omega_i(t) = \frac{d\theta(t)}{dt} = \omega_C + k_p \frac{m(t)}{dt} = \omega_C + k_p \dot{m}(t)$$

In phase modulation (PM) the instantaneous angular frequency ω_i varies linearly with the derivative of the message signal m(t) (denoted here by $\dot{m}$(t)).

k_p is phase-deviation (sensitivity) constant. Units: radians/volt
[Actually in radians/unit of the parameter $m(t)$.]

Definition: Instantaneous frequency is $\omega_i(t) = \frac{d\theta(t)}{dt}$

	Phase Modulation	Frequency Modulation
Angle	$\theta(t) = \omega_C t + k_p m(t)$	$\theta(t) = \omega_C t + k_f \int_{-\infty}^{t} m(\alpha)\, d\alpha$
Frequency	$\omega_i = \omega_C + k_p \frac{dm(t)}{dt}$	$\omega_i = \omega_C + k_f m(t)$

In phase modulation $m(t)$ drives the variation of phase θ.

In frequency modulation $m(t)$ drives the variation of frequency f.

Comparing Frequency Modulation to Phase Modulation

#	Frequency Modulation (FM)	Phase Modulation (PM)
1	Frequency deviation is proportional to modulating signal $m(t)$	Phase deviation is proportional to modulating signal $m(t)$
2	Noise immunity is superior to PM (and of course AM)	Noise immunity better than AM but not FM
3	Signal-to-noise ratio (SNR) is better than in PM	Signal-to-noise ratio (SNR) is not as good as in FM
4	FM is widely used for commercial broadcast radio (88 MHz to 108 MHz)	PM is primarily for some mobile radio services
5	Modulation index is proportional to modulating signal $m(t)$ as well as modulating frequency f_m	Modulation index is proportional to modulating signal $m(t)$

Phase and Frequency Deviation

Phase Deviation: In the waveform of Eq. A cos[ω_c t+Θ (t)] the maximum value attained by, that is, the maximum Phase Deviation of the total angle from the carrier angle ω_c t ,is called the Phase Deviation. $\Delta\Theta = \beta$

Frequency Deviation: The maximum change in instantaneous frequency ω_i from the average value i.e carrier frequency ω_c is known as **Frequency Deviation.**

The Frequency Deviation can be derived as

$$\omega_i = d\psi/dt = (d/dt)[\omega_c t + k_f \int m(t)dt]$$

$$\omega_i = \omega_c + k_f\ m(t)$$

$$\omega_i - \omega_c = k_f\ m(t)$$

The maximum change in instantaneous frequency ω_i from the average value ω_c will be depend on the magnitude and sign of k_f m (t).

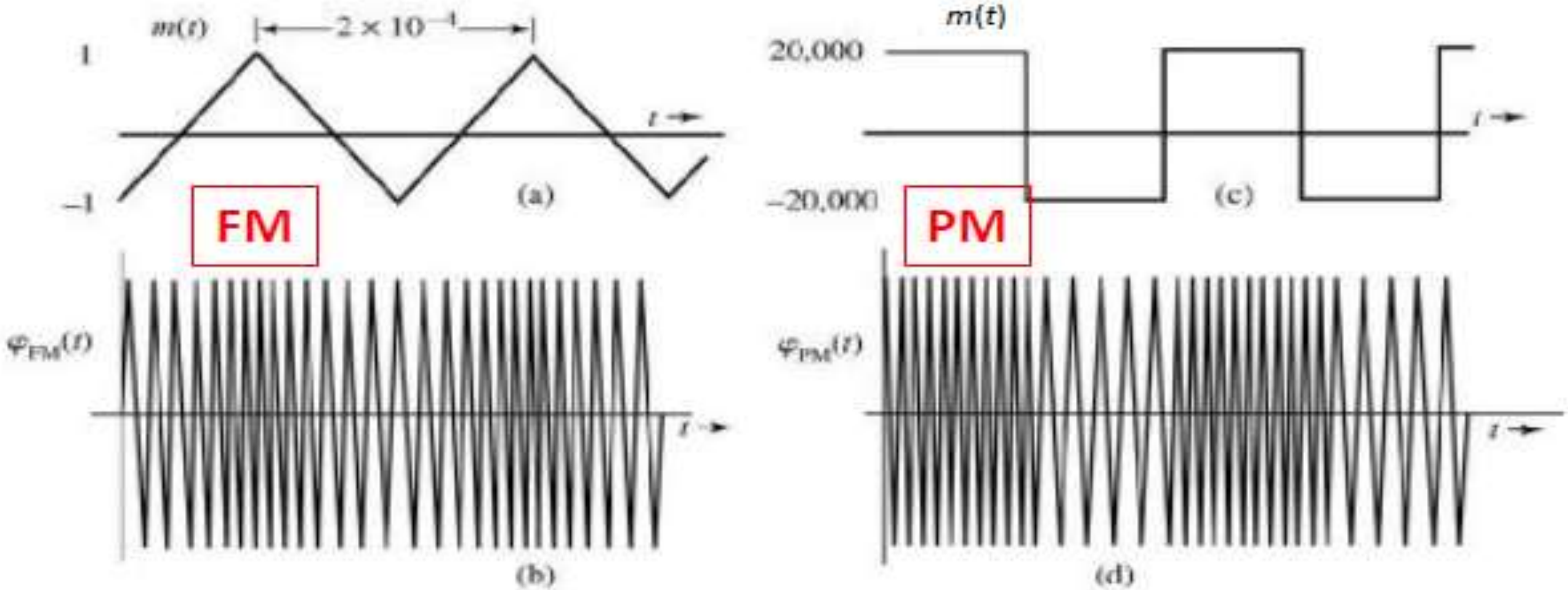
Thus. $\Delta\omega = |k_f\ m(t)|_{max}$

If m (t)= $E_m \cos \omega_m t$,so $\Delta\omega = |k_f\ E_m \cos \omega_m t|_{max}$

$|\cos \omega_m t|_{max} = 1$, $\Delta\omega = k_f\ E_m$

Frequency sensitivity $k_f = \Delta\omega / E_m$ radian per volt

Sketch FM and PM waveforms for the modulating signal $m(t)$. The constants k_f and k_p are $2\pi \times 10^5$ and 10π, respectively. Carrier frequency f_c = 100 MHz.



$$f_i = f_c + \frac{k_f}{2\pi} m(t) = 1\times10^8 + 1\times10^5 \cdot m(t);$$

$$m_{min} = -1 \text{ and } m_{max} = 1$$

$$(f_i)_{min} = 10^8 + 10^5(-1) = 99.9 \text{ MHz},$$

$$(f_i)_{max} = 10^8 + 10^5(+1) = 100.1 \text{ MHz}$$

$$f_i = f_c + \frac{k_p}{2\pi} \dot{m}(t) = 1\times10^8 + 5 \cdot \dot{m}(t);$$

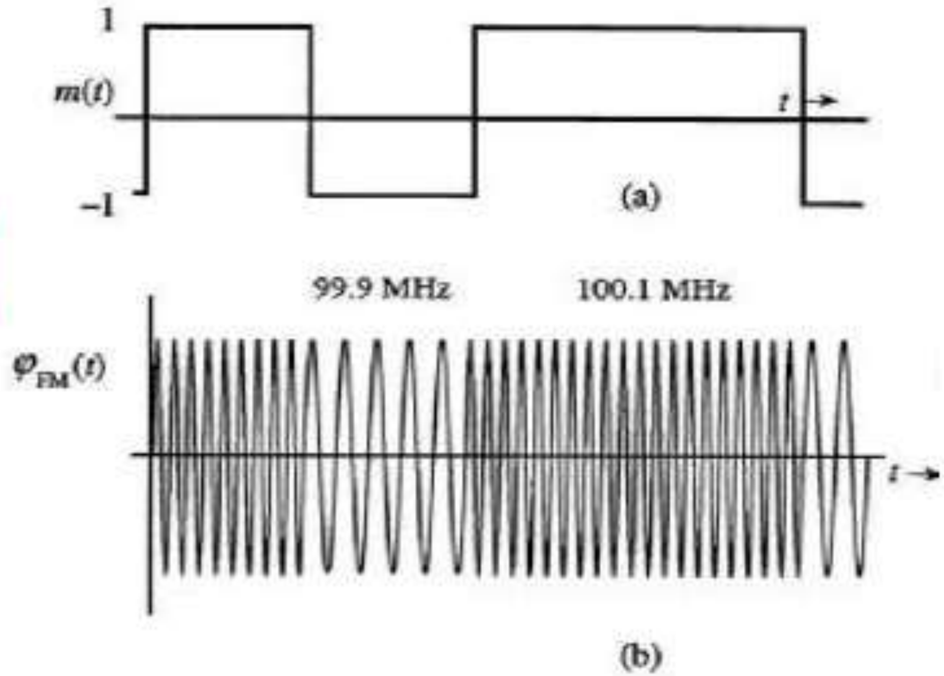
$$\dot{m}_{min} = -20,000 \text{ and } \dot{m}_{max} = 20,000$$

$$(f_i)_{min} = 10^8 + 5(-20,000) = 99.9 \text{ MHz},$$

$$(f_i)_{max} = 10^8 + 5(+20,000) = 100.1 \text{ MHz}$$

Sketch FM and PM waveforms for the modulating signal $m(t)$. The constants k_f and k_p are $2\pi \times 10^5$ and $\pi/2$, respectively. Carrier frequency $f_c = 100$ MHz.

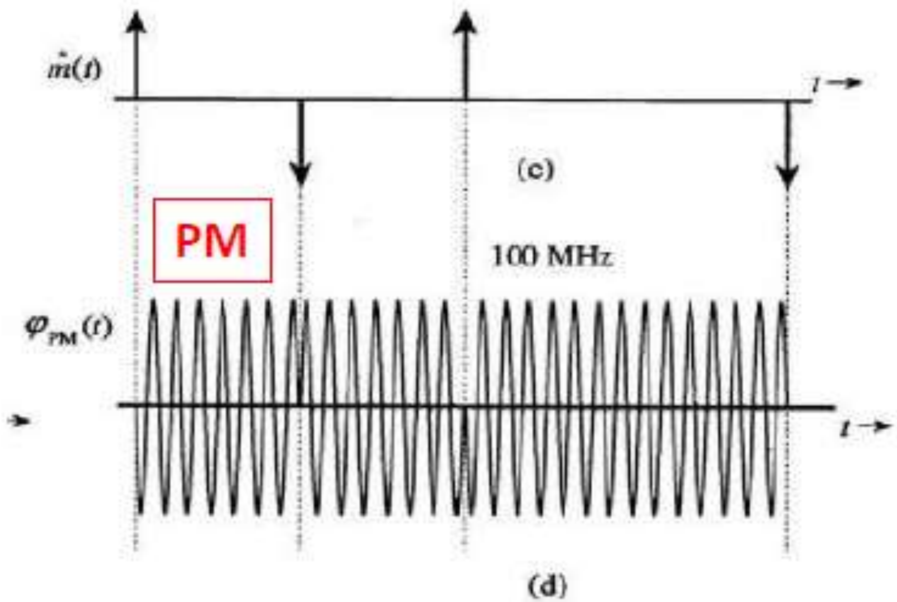
FM



$$f_i = f_C + \frac{k_f}{2\pi} m(t) = 1\times 10^8 + 1\times 10^5 m(t)$$

Since $m(t)$ switches from +1 to -1 and vice versa, the FM wave Frequency switches between 99.9 MHz and 100.1 MHz. This is called **Frequency Shift Keying (FSK)** and is a digital format.

Sketch FM and PM waveforms for the modulating signal $m(t)$. The constants k_f and k_p are $2\pi \times 10^5$ and $\pi/2$, respectively. Carrier frequency f_c = 100 MHz.



$$f_i = f_C + \frac{k_p}{2\pi}\dot{m}(t) = 1\times 10^8 + \frac{1}{4}\dot{m}(t)$$

This is carrier PM by a digital signal – it is **Phase Shift Keying (PSK)** because digital data is represented by phase of the carrier wave.

$$\varphi_{PM}(t) = A\cos\left[\omega_C t + k_p m(t)\right] = A\cos\left[\omega_C t + \frac{\pi}{2}m(t)\right]$$

$$\varphi_{PM}(t) = A\sin(\omega_C t) \quad \text{when} \quad m(t) = -1$$

$$\varphi_{PM}(t) = -A\sin(\omega_C t) \quad \text{when} \quad m(t) = 1$$

Types of Frequency Modulation

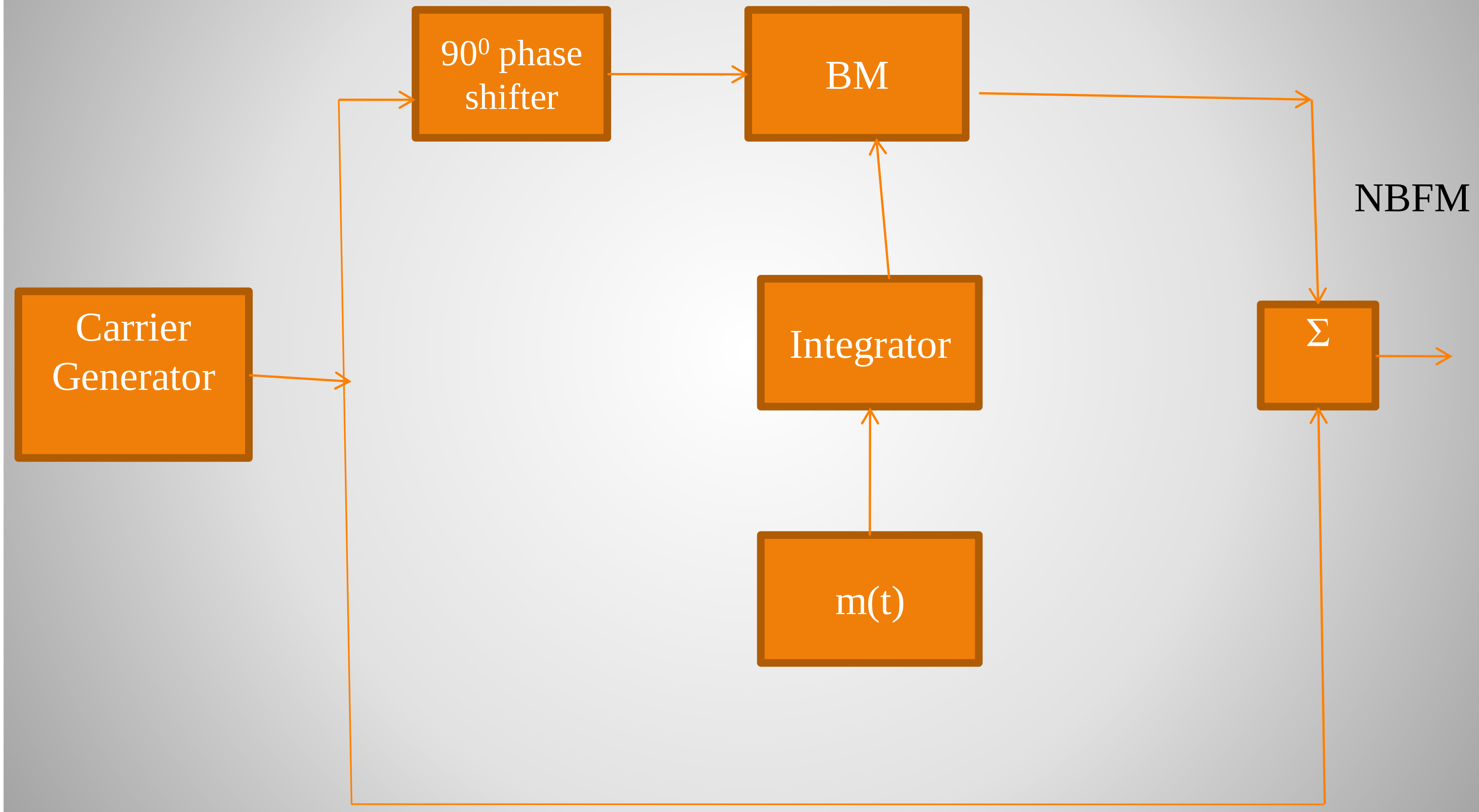
- The BW of an FM signal depends on Frequency Deviation k_f m (t).
- When deviation is high, the BW will be large and vice-versa.
- BW will depends on Frequency sensitivity k_f
- If k_f is too small then the BW will be narrow and vice-versa.
- The FM can be classified into two types depending upon frequency sensitivity k_f

- 1. Narrowband Frequency Modulation
- 2. Wide band Frequency Modulation

Narrowband Frequency Modulation

- When k_f is too small then the BW will be narrow and same as that of AM.
- The general expression for FM in the phasor form is given by
- $= Ae^{j[\omega c\ t + k_f\ m(t)]}$
- For NBFM, k_f g(t)<< 1 for all values of t
-
- $e^{j[k_f\ m(t)]} = 1+j\ k_f\ m(t)$
-
- So, $=\text{Re}\ \{A[1+j\ k_f\ m(t)]\ e^{j[\omega c\ t]}\}$
-
- $\varphi\ (t)= \text{Re}\{A[1+j\ k_f\ m(t)]\ [\cos \omega_c t +j \sin \omega_c t]\} =[A\cos \omega_c t - j\ A\ k_f\ m(t) \sin \omega_c t]$
-
- Similarly , NWPM is given by $\varphi\ (t)= [A\cos \omega_c t - j\ A\ k_f\ m(t) \sin \omega_c t]$
- The expression of FM and PM is similar to AM with slight modification.
- $m(t)= \int g(t)dt$
- F.T{ m(t)}=[G(ω)]/jω
- M(ω)=[G(ω)]/jω
- If M(ω) is band limited to ω_m,then G(ω) is also restricted to ω_m
- Both FM and PM expression have carrier and sideband terms similar to AM expression except with differing phase relations between carrier and sideband terms .this difference makes the amplitude of FM constant.

Generation of Narrowband FM:



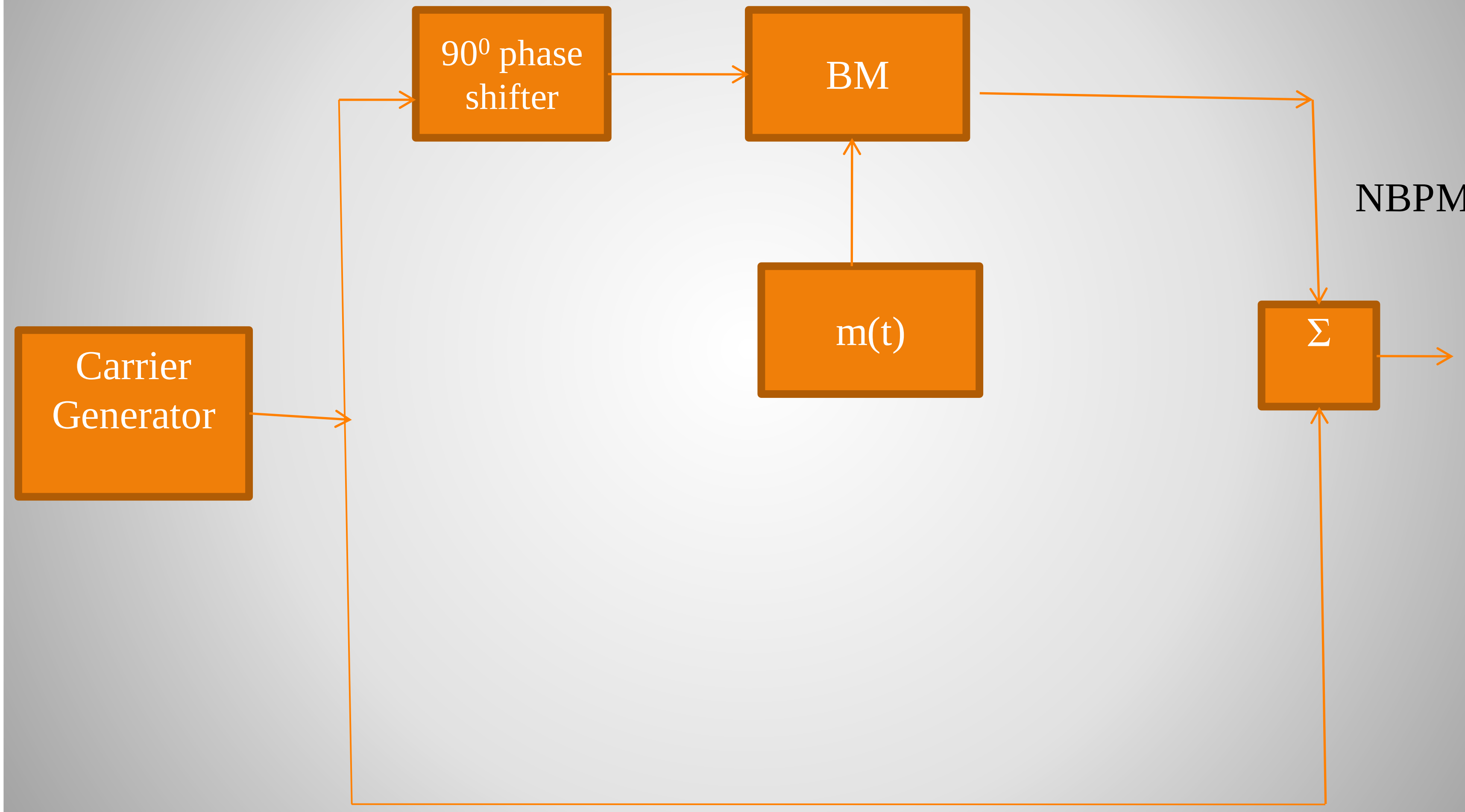
NBFM signal

- The modulating signal is given by $m(t)=E_m \cos \omega_m t$
- The carrier signal is given by $A \cos \omega_c t$
- The instantaneous frequency of the resulting modulated signal can be obtained by $\omega_i = \omega_c + k_f m(t) = \omega_c + k_f E_m \cos \omega_m t$
- Frequency Deviation $\Delta\omega = k_f E_m$
- $\omega_i = \omega_c + \Delta\omega \cos \omega_m t$
- The phase angle of the modulated signal can be obtained by integrating
- $\psi_i = \int \omega_i \, dt = \int [\omega_c + \Delta\omega \cos \omega_m t] dt$
- $\psi_i = [\omega_c t + (\Delta\omega/\omega_m) \sin \omega_m t]$
- $\psi_i = [\omega_c t + \beta \sin \omega_m t]$
- where $\beta = (\Delta\omega/\omega_m) = (k_f E_m/\omega_m)$
- β is known as modulation index of FM wave. It is a ratio of frequency deviation to the modulating frequency. The modulation index plays an important role in deciding the BW of FM system.

NBFM signal

- . The FM signal is given by $\varphi(t) = A\cos(\psi_i)$
- So, $\varphi(t) = A\cos(\omega_c t + \beta \sin \omega_m t)$
- The maximum change in the total phase angle from the center phase angle $\omega_c t$ is called the Phase Deviation. $\Delta\Theta = \beta$ radians
- Relationship between Phase and Frequency Deviation $\Delta\Theta = (\Delta\omega/\omega_m) = \beta$
- The modulation index β decides whether an FM is a NBFM or a WBFM because it is directly proportional to Frequency Deviation Δω.
- $\beta = 0.5$ is the transition point between NBFM and WBFM.
- The expression of NBFM is
- $\varphi(t) = A\cos\omega_c t - A k_f g(t) \sin\omega_c t$
- $g(t) = \int m(t)dt$
- $m(t) = E_m \cos\omega_m$
- $g(t) = \int E_m \cos\omega_m t\, dt = (E_m/\omega_m)\sin\omega_m t$
- So, $\varphi(t) = A\cos\omega_c t - A k_f (E_m/\omega_m)\sin\omega_m t \sin\omega_c t$
- $\varphi(t) = A\cos\omega_c t - A m_a \cos\omega_m t \cos\omega_c t$

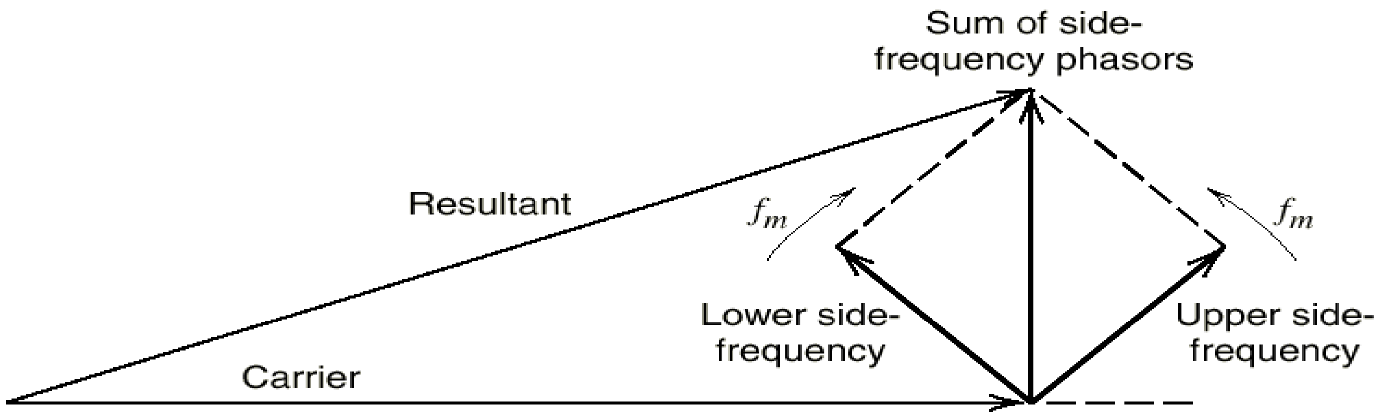
Generation of Narrowband PM:



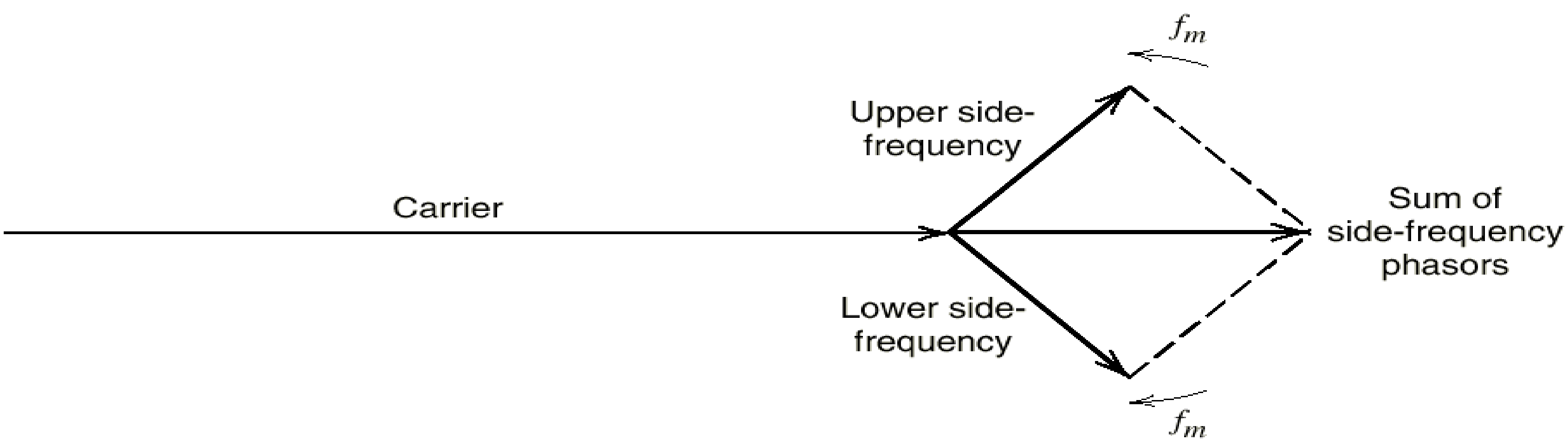
Phasor diagram of NBFM signal

- $\varphi_{AM}(t) = A\cos\omega_c t + A m_a \cos\omega_m t \cos\omega_c t$
- $\varphi_{FM}(t) = A\cos\omega_c t - A\beta \sin\omega_m t \sin\omega_c t$
- $\varphi_{AM}(t) = A\cos\omega_c t + (A m_a/2)[\cos(\omega_c + \omega_m)t + \cos(\omega_c - \omega_m)t]$
- $\varphi_{FM}(t) = A\cos\omega_c t + (A\beta/2)[\cos(\omega_c + \omega_m)t - \cos(\omega_c - \omega_m)t]$
- Consider a coordinate system rotating CCW at an angular frequency ω_c The carrier is fixed and is aligned in a horizontal direction.
- The sideband phasor rotate at an angular velocity ω_m relative to the carrier and in opposite directions to each other..
- In AM ,the resultant amplitude of the carrier varies as the sideband vector rotates.
- The phasor of the NBFM is shown in figure.The only difference is that the LSB phasor is reversed(opposite) as compared to the LSB phasor AM.
- The net resultant yieldsthe same amplitude as the unmodulated,i.e,OA'= OB'.The resultant of two sidebands in NBFM is alwayes perpendicular to carrier phasor,whereas in AM the resultant of two sidebands is alwayes parallel .

A phasor comparison of narrowband FM and AM waves for sinusoidal modulation. (a) Narrowband FM wave. (b) AM wave.



(a)



(b)

- Bandwidth of a sinusoidally modulated FM signal

Bandwidth of a sinusoidally modulated FM signal

Case II – Wideband FM (WBFM)

WBPM requires β >> 1 radian

For wideband FM we have a nonlinear process, with single tone modulation:

$$\varphi_{FM}^{WB}(t) = \mathrm{Re}\left[A_c \exp\left(j2\pi f_C t + j\beta \sin(2\pi f_m t)\right)\right]$$

We need to expand the exponential into a Fourier series so that we can analyze $\varphi_{FM}^{WB}(t)$.

$$\varphi_{FM}^{WB}(t) = A_C \sum_{n=-\infty}^{\infty} \mathrm{J}_n(\beta)\cdot\cos\left(2\pi(f_C + nf_m)t\right)$$

$$\beta = \frac{\Delta f}{f_m} = \frac{\Delta f}{B}$$

where the coefficients $\mathrm{J}_n(\beta)$ are Bessel functions.

Spectrum of an FM signal: Sinusoidal modulation Wideband FM

- When the β is large,the FM produces large no.s of sidebands and the BW of FM is quite large. Such system is called WBFM. will be narrow and same as that of AM.
- The general expression for FM in the phasor form is given by
- $\varphi FM = A\exp[j(\omega_c t + \beta \sin \omega_m t)] = A\exp(j\beta \sin \omega_m t)\ )\ e^{j[\omega c\, t]}$
- The term $A \exp(j\beta \sin \omega_m t)\ )$ is periodic function of period $1/f_m$ and can be expanded in the form of a complex Fourier series :

$$e^{j\beta \sin \omega_m t} = \sum_{n=-\infty}^{\infty} F_n e^{j\omega_m nt}$$

$$F_n = f_m \int_{-\pi/\omega_m}^{\pi/\omega_m} e^{j\beta \sin \omega_m t} e^{-jn\omega_m t} dt$$

$$F_n = (1/\pi) \int_{-\pi}^{\pi} e^{j(\beta \sin x - nx)} dx$$

$$F_n = J_n(\beta)$$

The integration of R.H.S is recognized as the nth order of Bessel Function of the first kind and argument J(β)

$$e^{j\beta \sin \omega_m t} = \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j\omega_m nt}$$

$$\tilde{\varphi}_{FM}(t) = A e^{j\omega_c t} \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j\omega_m nt} = A \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j(\omega_c - n\omega_m)t}$$

$$\varphi_{FM}(t) = A \sum_{n=-\infty}^{\infty} J_n(\beta) \cos(\omega_c + n\omega_m)t$$

$$e^{j\beta \sin \omega_m t} = \sum_{n=-\infty}^{\infty} F_n e^{j\omega_m n t}$$

$$F_n = f_m \int_{-\pi/\omega_m}^{\pi/\omega_m} e^{j\beta \sin \omega_m t} e^{-jn\omega_m t} dt$$

$$F_n = (1/\pi) \int_{-\pi}^{\pi} e^{j(\beta \sin x - nx)} dx$$

$$F_n = J_n(\beta)$$

The integration of R.H.S is recognized as the nth order of Bessel Function of the first kind and argument J(β)

$$e^{j\beta \sin \omega_m t} = \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j\omega_m n t}$$

$$\tilde{\varphi}_{FM}(t) = A e^{j\omega_c t} \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j\omega_m n t} = A \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j(\omega_c - n\omega_m)t}$$

$$\varphi_{FM}(t) = A \sum_{n=-\infty}^{\infty} J_n(\beta) \cos(\omega_c + n\omega_m)t$$

Bessel Function

- The Bessel function $J_n(\beta)$ can be expanded in a power series given by

$$J_n(\beta) = \sum_{m=0}^{\infty} \frac{(-1)^m (\beta/2)^{n+2m}}{m!(n+m)!}$$

Some features of the Bessel coefficients

1. $J_n(\beta) = (-1)^n J_{-n}(\beta)$

2. If β is small, then $J_0(\beta) \approx 1,$

$$J_1(\beta) \approx \frac{\beta}{2},$$

$$J_n(\beta) \approx 0 \quad \text{for all} \quad n > 2$$

3. $\sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1$

- WideBand FM

- By using the first property of Bessel function the FM signal is given by
- $\varphi_{FM}(t) =) = AJ_0(\beta)\cos(\omega_c t) - AJ_1(\beta)[\cos(\omega_c - \omega_m)t - \cos(\omega_c + \omega_m)t] + A J_2(\beta)\,)[\cos(\omega_c - 2\omega_m)t + \cos(\omega_c + 2\omega_m)t] - AJ_3(\beta)\,)[\cos(\omega_c - 3\omega_m)t - \cos(\omega_c + 3\omega_m)t] + \ldots\ldots\ldots\ldots$

- The following observations are made from the above equation:
- 1.Frequency Components :(a) the FM signal has a carrier term cos $\omega_c t$ with magnitude $AJ_0(\beta)$.The maximum value of $J_0(\beta)$ is 1 when $\beta = 0$.(No modulation condition).the carrier term is absent at $\beta = 2.4, 5.52$and so on for which $J_0(\beta)$ is zero. Therefore the magnitude of carrier is reduced by a factor $J_0(\beta)$.
- (b) Theoretically infinite no.s of sidebands are produced and the amplitude of each sideband is dependent upon the corresponding Bessel function

$J_n(\beta)$.The sidebands with small amplitudes are ignored. The sidebands having considerable amplitudes are known as significant sidebands. They are finite in numbers.

- WideBand FM

- The following observations are made from the above equation:
- 2.Narrowband FM :(a) For a small value of β(less than 0.6),only $J_0(\beta)$ and $J_1(\beta)$ are significant, and the higher terms are negligible. Thus, FM signal has a carrier term NBFM.

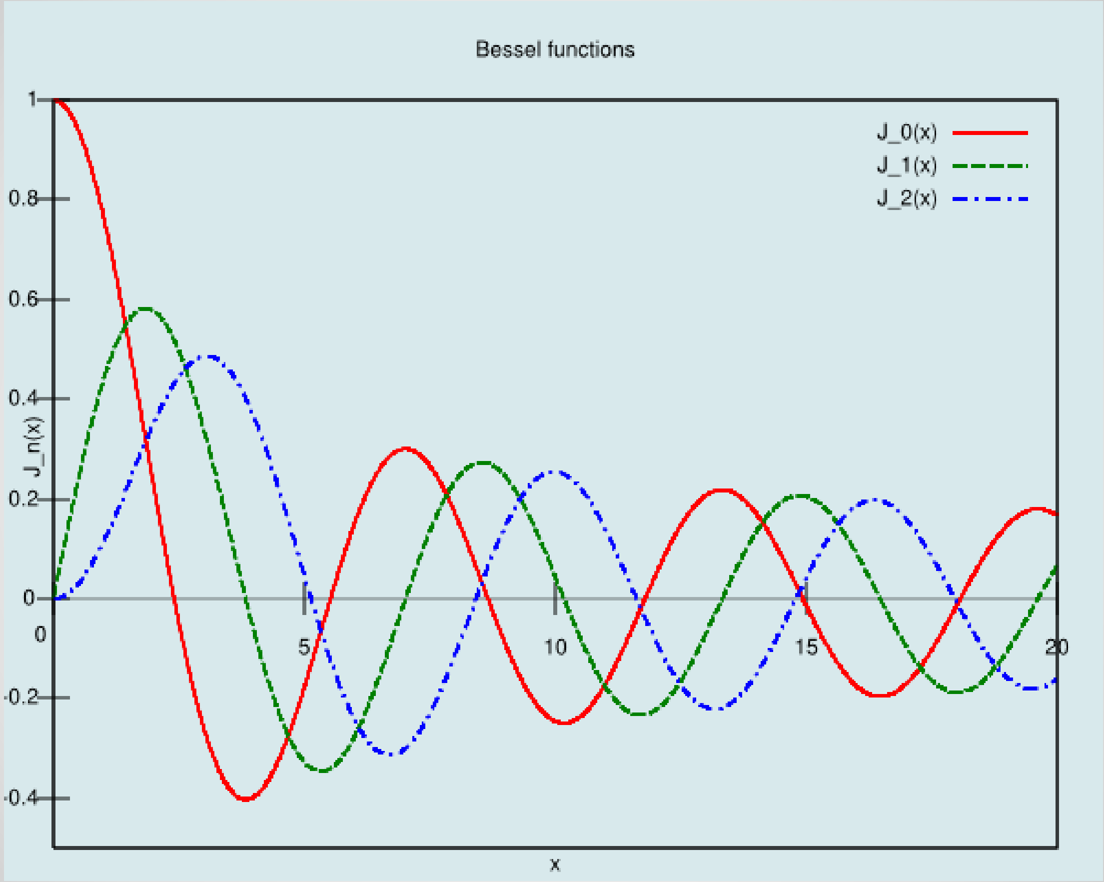
3.Power content in FM signal: Since the amplitude of FM remains unchanged, the power of the FM signal is same as that of unmodulated carrier.The Parseval's theorem sates that the total power of the signal is equal to the sum of the power of individual components present in it.beacuse the summation of in below equation yields unity.Thus the FM power is same as that of the unmodulated carrier.

$$\overline{\varphi_{FM}^2} = \frac{A^2}{2} \sum_{k=-\infty}^{\infty} J_n^2(\beta) = \frac{A^2}{2}$$

4.Transmission efficiency:

- WideBand FM

4.Transmission efficiency: In FM, out of the total power $A^2/2$,the power caried by the carrier term depends on the value of $J_0(\beta)$, and the power carried by a sideband depends on the value of corresponding $J_0(\beta)$ .If we adjust β 2.4,5.52, or any such value so that, $J_0(\beta) = 0$, then the power carried by the carrier term in the FM signal will be zero. All the power being carricd by the sidebands provides a 100per cent transmission efficiency. For other values of β some power is carried by the carrier also, and efficiency is less than 100 per cent. Therefore, by adjusting the value of β we can get FM efficiency much more than AM is 33%, and may approach equal to the efficiency of AM-SC (i.c., 100 %) . Thus, FM has the efficiency between AM and SC



Bessel functions
J_0(x)
J_1(x)
J_2(x)
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
J_n(x)
0
5
10
15
20
x

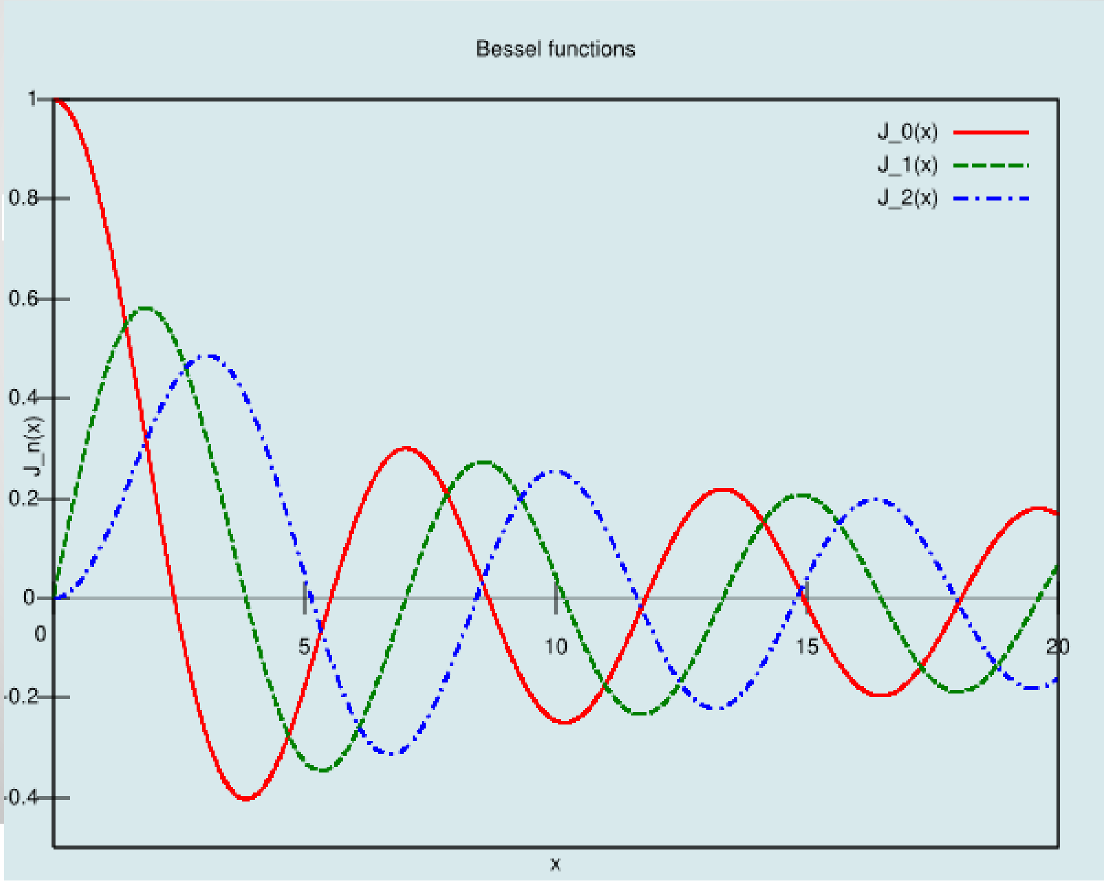
- WideBand FM

- The FM signal is given by $\varphi(t)=\cos(\omega_c t + \beta \sin \omega_m t)$
- $\varphi(t)= \cos(\omega_c t) \cos(\beta\sin \omega_m t) - \sin(\omega_c t)\sin(\beta\sin \omega_m t)$
- $\cos(\beta\sin \omega_m t)=J_0(\beta) +2 J_2(\beta) \cos(2\omega_m t) +2 J_4(\beta) \cos(4\omega_m t)+\ldots\ldots+$
- $+2 J_{2n}(\beta) \cos(2n\omega_m t)+\ldots\ldots$
- $\sin(\beta\sin \omega_m t)=2 J_1(\beta)\sin(\omega_m t) +2 J_3(\beta)\sin(3\omega_m t)+\ldots\ldots+ +2 J_{2n-1}(\beta) \sin(2n-1)\omega_m t +\ldots\ldots$
- The functions $J_n(\beta)$ are known as Bessel functions of first kind and order of n.
- The FM signal $\varphi(t)$ becomes
- $\varphi(t)= \cos(\omega_c t)[J_0(\beta) +2 J_2(\beta) \cos(2\omega_m t) +2 J_4(\beta) \cos(4\omega_m t)+\ldots\ldots+ +2 J_{2n}(\beta) \cos(2n\omega_m t)+\ldots\ldots] - \sin(\omega_c t)[2 J_1(\beta)\sin(\omega_m t) +2 J_3(\beta)\sin(3\omega_m t)+\ldots\ldots+ 2 J_{2n-1}(\beta) \sin(2n-1)\omega_m t +\ldots\ldots]$
- $\varphi(t)= J_0(\beta) \cos(\omega_c t) - J_1(\beta)[\cos(\omega_c -\omega_m)t - \cos(\omega_c +\omega_m)t]+ J_2(\beta))[\cos(\omega_c -2\omega_m)t + \cos(\omega_c +2\omega_m)t]- J_3(\beta))[\cos(\omega_c -3\omega_m)t - \cos(\omega_c +3\omega_m)t]+\ldots\ldots\ldots\ldots$
- $\varphi_{FM}(t)= A$

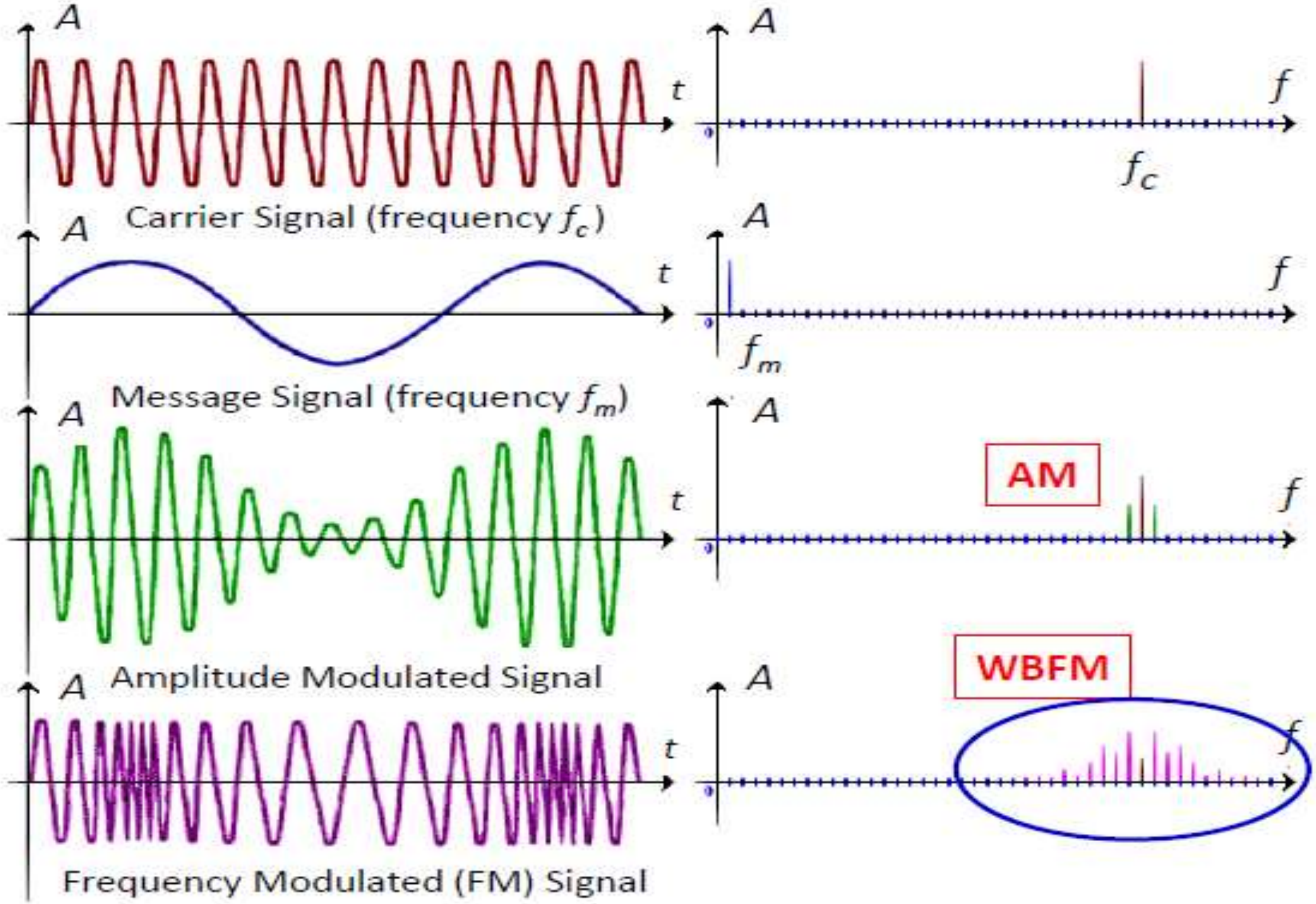
Some features of the Bessel coefficients

- f: is the frequency of the signal to be transmitted
- The minimum antenna height required to transmit a baseband signal of $f = 3$ kHz is calculated as follows : $\lambda/4 = 25$KM.The antenna of this height is practically impossible to install .

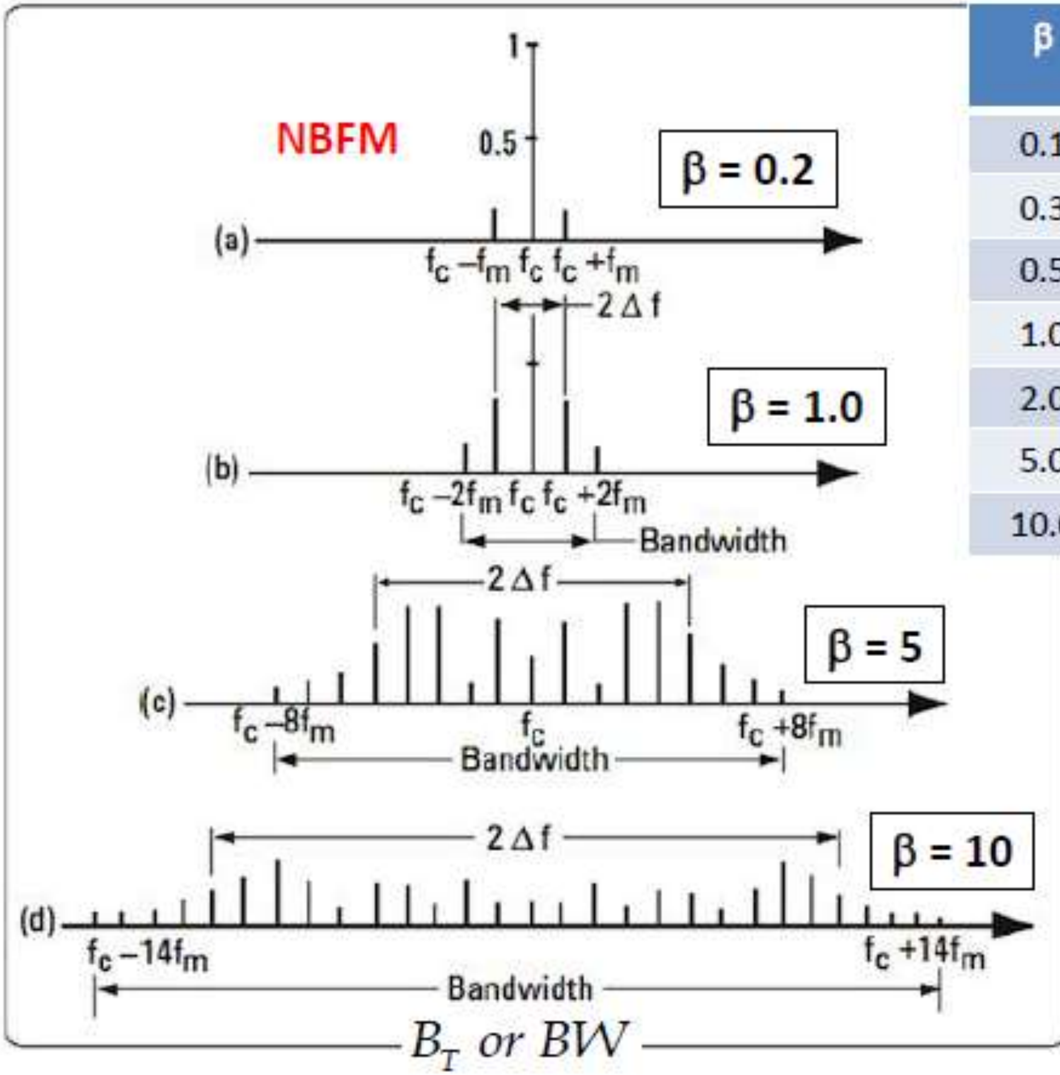
Effect of the modulation index β on bandwidth



FM (or PM) Requires Much More bandwidth Than AM



FM Spectra as Function of Modulation Index β



β	Number of Sidebands¶	Bandwidth
0.1	2	$2f_m$
0.3	4	$4f_m$
0.5	4	$4f_m$
1.0	6	$6f_m$
2.0	8	$8f_m$
5.0	16	$16f_m$
10.0	28	$28f_m$

Single tone modulation

$$\beta = \frac{\Delta f}{f_m}$$

Angle Modulation: An Example

- An angle-modulated signal with carrier frequency $\omega_c = 2\pi \times 10^6$ is described by the equation:

$$\phi_{EM}(t) = 12\cos(\omega_c t + 5\sin 1500t + 10\sin 2000\pi t)$$

1. Determine the power of the modulating signal.
2. What is Δf?
3. What is β?
4. Determine $\Delta\phi$, the phase deviation.
5. Estimate the bandwidth of $\phi_E M(t)$?

1. $P = 12^2/2 = 72\ units$
2. Frequency deviation Δf, we need to estimate the instantaneous frequency:

$$\omega_i = \frac{d}{dt}\theta(t) = \omega_c + 7,500\cos 1500t + 20,000\pi t$$

The deviation of the carrier is $\Delta\omega = 7,500\cos 1500t + 20,000\pi t$. When the two sinusoids add in phase, the maximum value will be $7,500 + 20,000\pi$ Hence $\Delta f = \frac{\Delta\omega}{2\pi} = 11,193.66Hz$

3. $\beta = \frac{\Delta f}{B} = \frac{11,193.66}{1000} = 11.193$
4. The angle $\theta(t) = \omega_c t + 5\sin 1500t + 10\sin 2000\pi t)$. The maximum angle deviation is 15, which is the phase deviation.
5. $B_{EM} = 2(\Delta f + B) = 24,387.32\ Hz$

- A single tone FM signal is

$$\varphi_{FM}(t) = 10\left[\cos\left(2\pi(10^6)t + 8\sin(2\pi(10^3)t)\right)\right]$$

Determine

a) the carrier frequency f_c

b) the modulation index β

c) the peak frequency deviation Δf

d) the bandwidth of $\varphi_{FM}(t)$

Solution to Example

Start with the basic FM equation:

$$\varphi_{FM}(t) = A_C\left[\cos\left(2\pi f_C t + \beta\sin(2\pi f_m t)\right)\right]$$

Compare this to

$$\varphi_{FM}(t) = 10\left[\cos\left(2\pi(10^6)t + 8\sin(2\pi(10^3)t)\right)\right]$$

(a) We see by inspection that f_c = 1,000,000 Hz and f_m = 1000 Hz.

(b) The modulation index is β = 8.

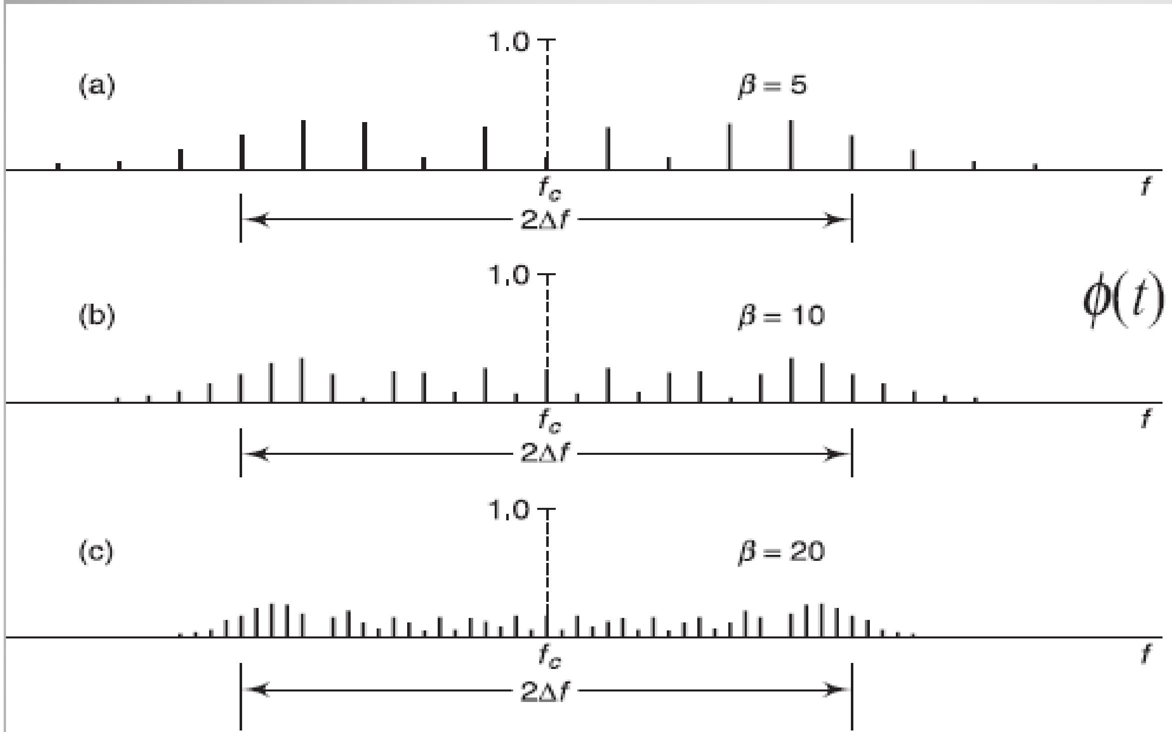
(c) The peak deviation frequency Δf is

$$\Delta f = \beta \cdot f_m = 8 \cdot 1000 = 8{,}000\text{ Hz}$$

(d) The bandwidth is

$$B_{FM} = 2f_m(\beta+1) = 2{,}000(8+1) = 18{,}000\text{ Hz}$$

Spectrum of "constant bandwidth" FM



$$\phi(t) = (k/2\pi f_m)v_m \sin 2\pi f_m t$$

$$\beta = \frac{k v_m}{2\pi f_m}$$

bandwidth is $B \cong (2k/2\pi)v_m$, independently of f_m.

nominal bandwidth $B \cong 2\ \Delta f = 2\beta f_m$

Transmission Bandwidth of FM waves:

An FM wave consists of infinite number of side bands so that the bandwidth is theoretically infinite. But, in practice, the FM wave is effectively limited to a finite number of side band frequencies compatible with a small amount of distortion. There are many ways to find the bandwidth of the FM wave.

1. Carson's Rule: In single–tone modulation, for the smaller values of modulation index the bandwidth is approximated as 2fm. For the higher values of modulation index, the bandwidth is considered as slightly greater than the total deviation $2\Delta f$. Thus the Bandwidth for sinusoidal modulation is defined as:

For non-sinusoidal modulation, a factor called Deviation ratio (D) is considered. The deviation ratio is defined as the ratio of maximum frequency deviation to the bandwidth of message signal.

Deviation ratio , $D = (\Delta f / W)$, where W is the bandwidth of the message signal and the corresponding bandwidth of the FM signal is,

$$BT = 2(D + 1) W$$

2. Universal Curve : An accurate method of bandwidth assessment is done by retaining the maximum number of significant side frequencies with amplitudes greater than 1% of the unmodulated carrier wave. Thus the bandwidth is defined as “the 99 percent bandwidth of an FM wave as the separation between the two frequencies beyond which none of the side-band frequencies is greater than 1% of the carrier amplitude obtained when the modulation is removed”.

Transmission Bandwidth - $BW = 2\, n_{max} f_m$, (5.26)

where f_m is the modulation frequency and ‘n’ is the number of pairs of side-frequencies such that □Jn(□)□> 0.01. The value of n_{max} varies with modulation index and can be determined from the Bessel coefficients. The table 5.2 shows the number of significant side frequencies for different values of modulation index.

The transmission bandwidth calculated using this method can be expressed in the form of a universal curve which is normalised with respect to the frequency deviation and plotted it versus the modulation index. (Refer fig-5.7).

Table 5.2

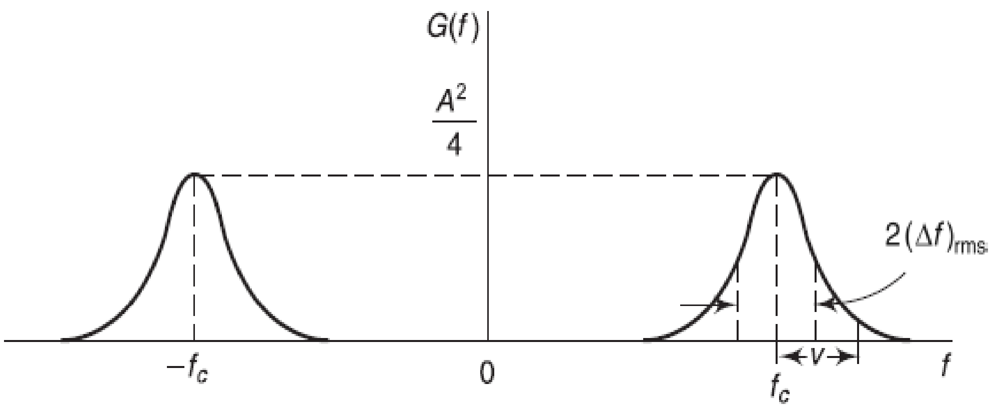
From the universal curve, for a given message signal frequency and modulation index the ratio (B/ Δf) is obtained from the curve. Then the bandwidth is calculated as:

□ □ 27.5...)()(m TT T f f B f f B B □ □□ □ □ □

Bandwidth required for a Gaussian modulated WBFM signal

Since $m(t)$ is Gaussian, so also is $G(f)$

$$G(f) = \frac{A^2}{4\sqrt{2\pi}\,\Delta f_{\text{rms}}}\left[e^{-(f-f_c)^2/2(\Delta f_{\text{rms}})^2} + e^{-(f+f_c)^2/2(\Delta f_{\text{rms}})^2} \right]$$



Bandwidth calculation:

$$\frac{1}{\sqrt{2\pi}\,\Delta f_{\text{rms}}}\int_{-B/2}^{B/2} e^{-v^2/2(\Delta f_{\text{rms}})^2}\, dv = 0.98$$

$$\frac{2}{\sqrt{\pi}}\int_{0}^{B/(2\sqrt{2}\Delta f_{\text{rms}})} e^{-x^2}\, dx = 0.98$$

$$\operatorname{erf}\frac{B}{2\sqrt{2}\,\Delta f_{\text{rms}}} = 0.98$$

$$B = 2\sqrt{2}\,(1.645)\,\Delta f_{\text{rms}} = 4.6\,\Delta f_{\text{rms}}$$

Bandwidth of WBFM

Rule of thumb : If individual modulating signals alone produce deviations $(\Delta f)_1, (\Delta f)_2$, etc. then a pessimistic approach gives $B = 2[(\Delta f)_1 + (\Delta f)_2$

and an optimistic approach gives $B = 2[\text{rms value of } (\Delta f)\text{'s}]$

When the modulating signal has continuous spectral density and gives rise to FM with sidebands having continuous spectral density, bandwidth definition often used is

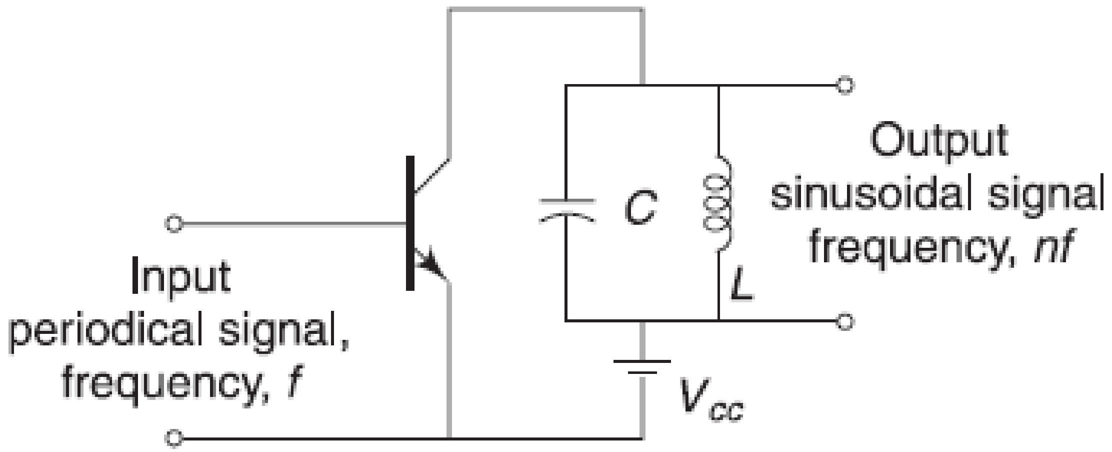
$$B \equiv 2\left[\frac{\int_{-\infty}^{\infty} \nu^2 G(\nu)\, d\nu}{\int_{-\infty}^{\infty} G(\nu)\, d\nu}\right]^{1/2}$$

Frequency Multiplication Applied to FM signal

A frequency multiplier is a combination of nonlinear element and band pass filter. This can be used to increase the deviation of FM.

- The frequency multiplier consists of a nonlinear device followed by a band-pass filter. The nonlinear device used is a memory less device. If the input to the nonlinear device is an FM wave with frequency, fc and deviation, Δf1 then its output v(t) will consist of dc component and 'n' frequency modulated waves with carrier frequencies, fc, 2fc, 3fc, …… nfc and frequency deviations a Δf1, 2Δf1 , 3Δf1 , ... nΔf1 respectively.
- The band pass filter is designed in such a way that it passes the FM wave centered at the frequency, nfc with frequency deviation nΔf1 and to suppress all other FM components. Thus the frequency multiplier can be used to generate a wide band FM wave from a narrow band FM wave.

Frequency Multiplication Applied to FM signal

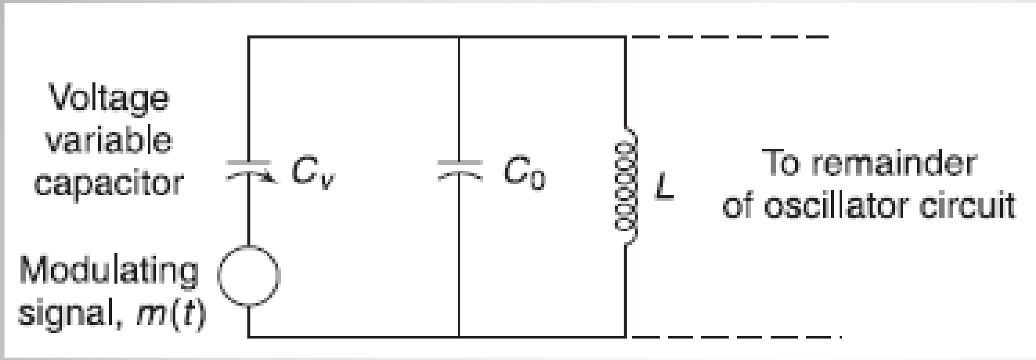


A frequency-multiplier circuit.

Frequency Multiplier

- The frequency multiplier consists of a nonlinear device followed by a band-pass filter. The nonlinear device used is a memory less device. If the input to the nonlinear device is an FM wave with frequency, fc and deviation, $\Delta f1$ then its output v(t) will consist of dc component and ‘n’ frequency modulated waves with carrier frequencies, fc, 2fc, 3fc, …… nfc and frequency deviations a $\Delta f1$, $2\Delta f1$, $3\Delta f1$, ... $n\Delta f1$ respectively.
- The band pass filter is designed in such a way that it passes the FM wave centered at the frequency, nfc with frequency deviation $n\Delta f1$ and to suppress all other FM components. Thus the frequency multiplier can be used to generate a wide band FM wave from a narrow band FM wave.

FM Generation : Parameter variation method



$$f = (2\pi\sqrt{LC})^{-1}$$

Voltage Controlled Oscillator (VCO) :

$$v_{osc}(t) = B\cos\left[\omega_c t + G_0 \int_{-\infty}^{t} v(\lambda)d\lambda\right]$$

$$\omega_i(t) = \omega_c + G_0 v(t)$$

The principal difficulty here is to maintain a stable carrier Frequency over extended period of time.

FM generation: Parameter-variation method

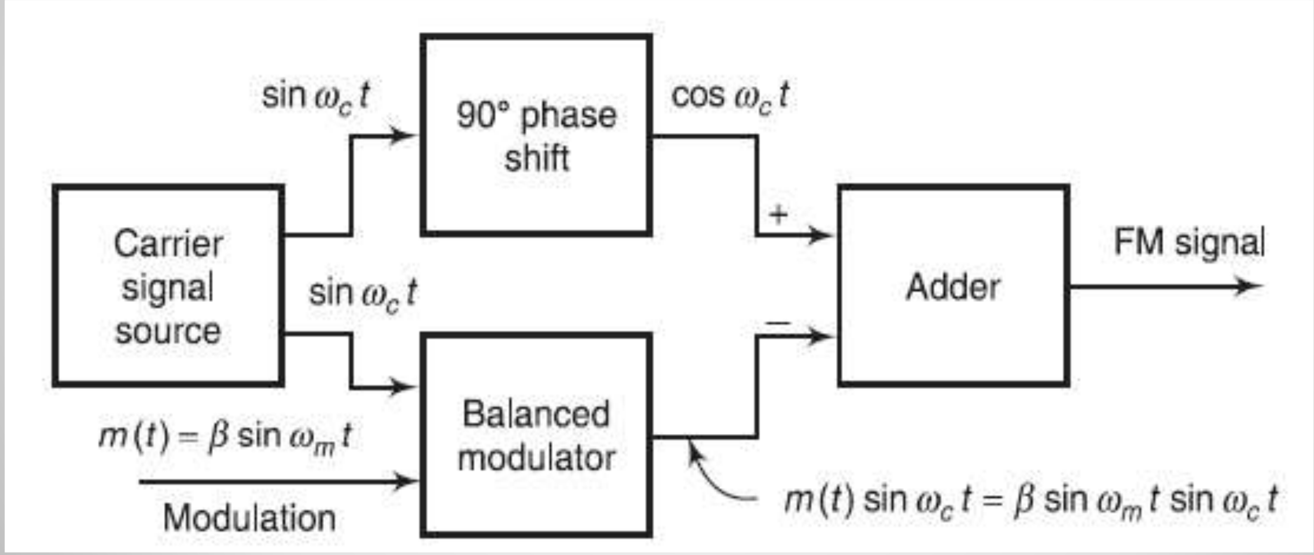
Generation of WBFM using Indirect Method:

- In in fff method a NBFM wave is generated first and frequency multiplication is next used to increase the frequency deviation to the desired level. The narrow band FM wave is generated using a narrow band phase modulator and an oscillator. The narrow band FM wave is then passed through a frequency multiplier to obtain the wide band FM wave, as shown in the fig:(5.9). The crystal controlled oscillator provides good frequency stability. But this scheme does not provide both the desired frequency deviation and carrier frequency at the same time. This problem can be solved by using multiple stages of frequency multiplier and a mixer stage.

FM Generation : Armstrong's indirect method

for $|m(t)| \ll 1$,

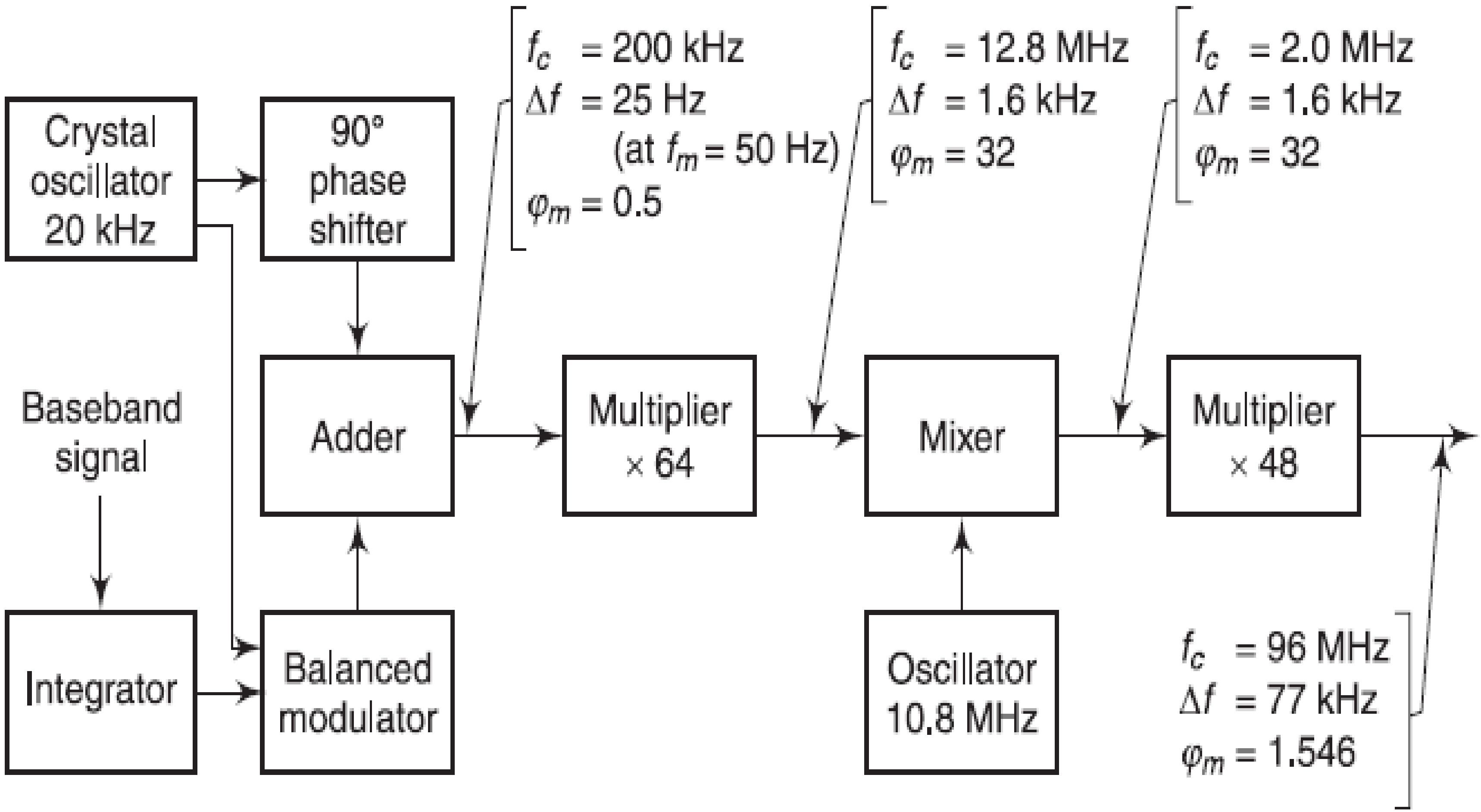
$$\cos[\omega_c t + m(t)] \cong \cos \omega_c t - m(t) \sin \omega_c t$$



The difficulty with parameter variation method is avoided.
Crystal is the carrier source.

- Generation of WBFM by Armstrong’s Method:
- Armstrong method is an indirect method of FM generation. It is used to generate FM signal having both the desired frequency deviation and the carrier frequency. In this method, two-stage frequency multiplier and an intermediate stage of frequency translator is used, as shown in the fig:(5.10). The first multiplier converts a narrow band FM signal into a wide band signal. The frequency translator, consisting of a mixer and a crystal controlled oscillator shifts the wide band signal to higher or lower frequency band. The second multiplier then increases the frequency deviation and at the same time increases the center frequency also. The main design criteria in this method are the selection of multiplier gains and oscillator frequencies. This is explained in the following steps.

Example of an Armstrong FM system



Design Steps: How to choose n1 and n2 for the given specifications?

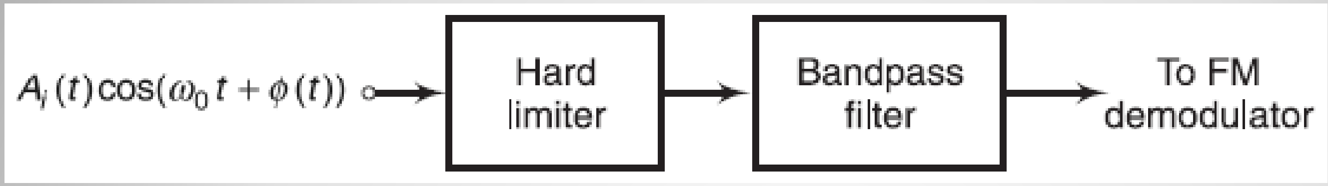
- Step1. Select the value of $\beta < 0.5$ for the narrow band phase modulator. This value limits the harmonic distortion by NBPM to minimum.
- Step 2. The requirement is that the frequency deviation produced by the lowest modulation frequencies is raised to required Δf. So choose the frequency deviation of NBFM, Δf_1 by selecting the minimum value of f_m
- $\Delta f_1 = \beta f_m$ (min). ---- (a)
- Step 3. Frequency Multipliers change the frequency deviation. Hence the total change in the frequency deviation is product of the two deviations:
- $n_1.n_2 = \Delta f / \Delta f_1$ ----- (b)
- Step 4. Frequency Translator (mixer & oscillator) will not change the frequency deviation, it only shifts the FM signal to either upwards and downwards in the spectrum. The output of mixer is
- For down ward translation: $f_c = n_2 (f_2 - n_1 . f_1)$ ---- (c)
- and for upward translation: $f_c = n_2 (n_1 . f_1 - f_2)$.
- Step 5. Choose suitable value for f_2 and solve the equations (b) and (c) simultaneously to find the multiplying factors n_1 and n_2.
-

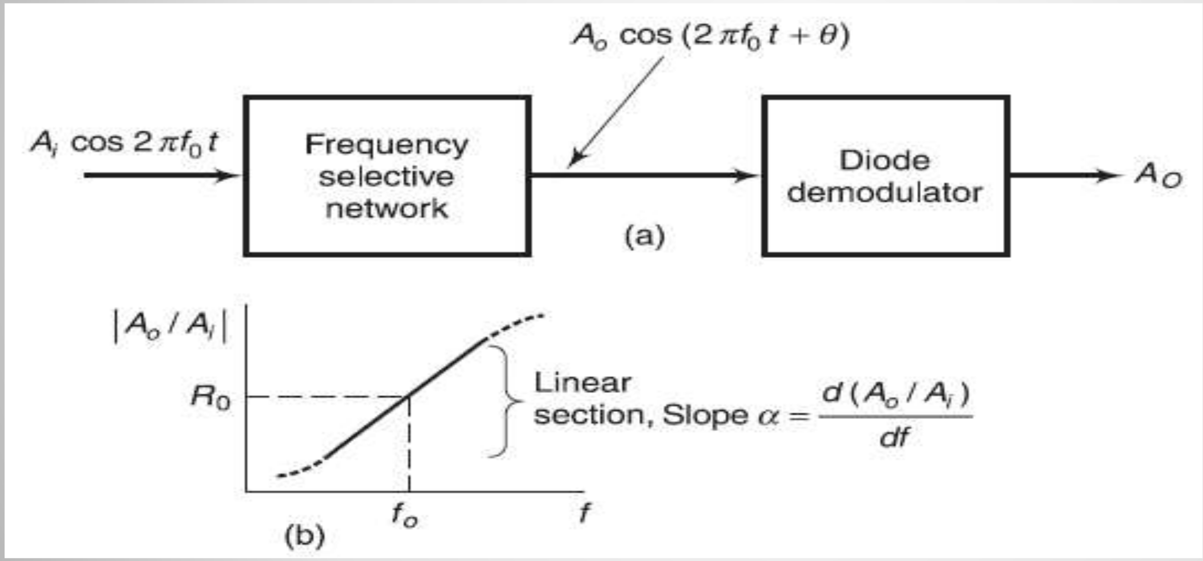
- Where
- Thus the instantaneous frequency fi(t) is defined as: 4.5...2cos)(0 t ffftf m i □ □□□□
- The term, Δf represents the frequency deviation and the relation with Δc is given by: Thus the output of the oscillator will be an FM wave. But the direct method of generation has the disadvantage that the carrier frequency will not be stable as it is not generated from a highly stable oscillator.
- Generally, in FM transmitter the frequency stability of the modulator is achieved by the use of an auxiliary stabilization circuit as shown in the fig.(5.12). Fig: 5.12 – Frequency stabilized FM modulator.
- The output of the FM generator is applied to a mixer together with the output of crystal controlled oscillator and the difference is obtained. The mixer output is applied to a frequency discriminator, which gives an output voltage proportional to the instantaneous frequency of the FM wave applied to its input. The discriminator is filtered by a low pass filter and then amplified to provide a dc voltage. This dc voltage is applied to a voltage controlled oscillator (VCO) to modify the frequency of the oscillator of the FM generator. The deviations in the transmitter carrier frequency from its assigned value will cause a change in the dc voltage in a way such that it restores the carrier frequency to its required

Advantages and disadvantages of FM over AM:

- Advantages of FM over AM are:
- 1. Less radiated power.
- 2. Low distortion due to improved signal to noise ratio (about 25dB) w.r.t. to man made interference.
- 3. Smaller geographical interference between neighboring stations.
- 4. Well defined service areas for given transmitter power.
- Disadvantages of FM:
- 1. Much more Bandwidth (as much as 20 times as much).
- 2. More complicated receiver and transmitter.
- Applications:
- Some of the applications of the FM modulation are listed below:
- I. FM Radio,(88-108 MHz band, 75 kHz,)
- II. TV sound broadcast, 25 kHz,
- III. 2-way mobile radio, 5 kHz / 2.5 kHz.

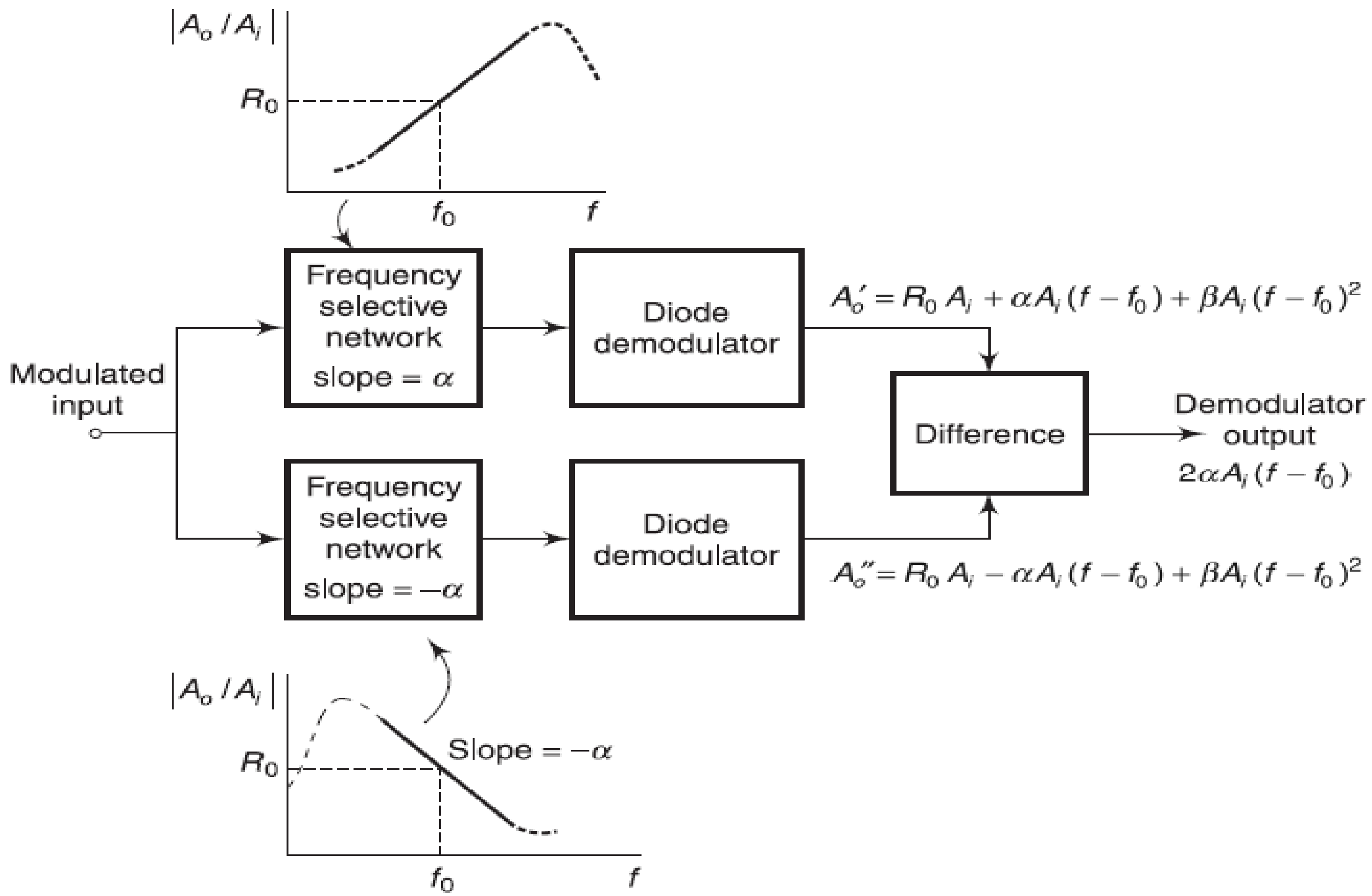
FM demodulators





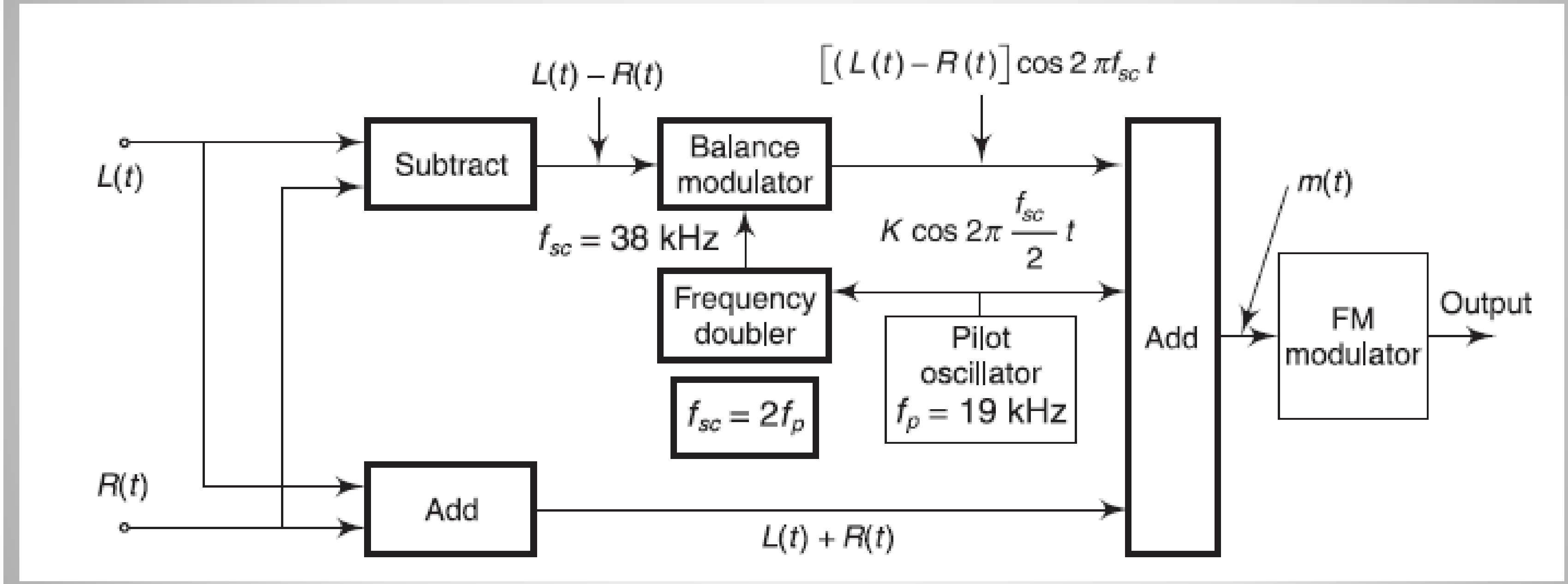
Frequency selective network typically is an *LC* circuit.

Balanced FM Demodulator



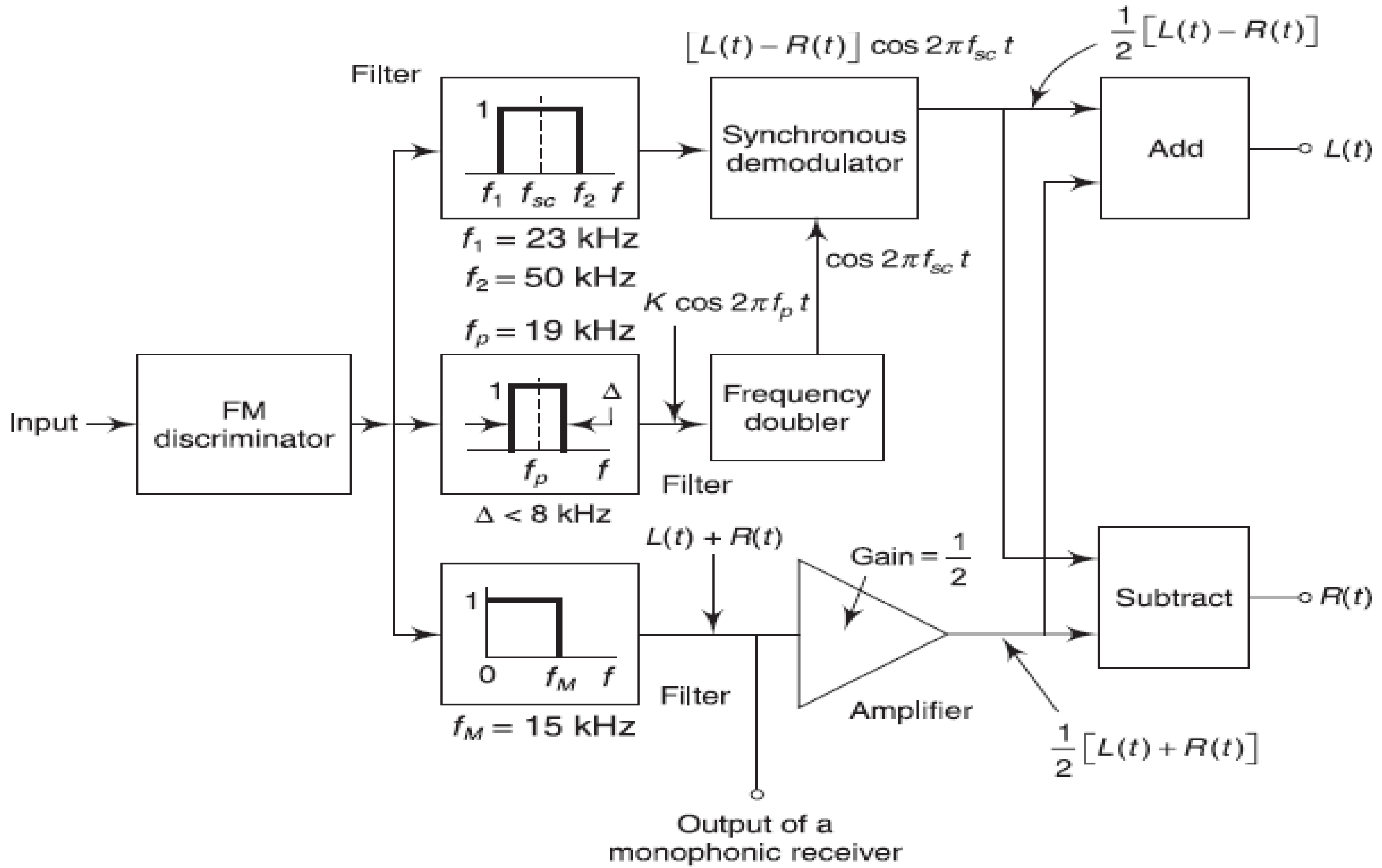
Stereophonic FM broadcasting

In this, two microphones are used, two audio baseband signals are transmitted. Received signals are played on two speakers.



Stereophonic Transmitter

Stereophonic Receiver



THANKS

ANY QUERIES